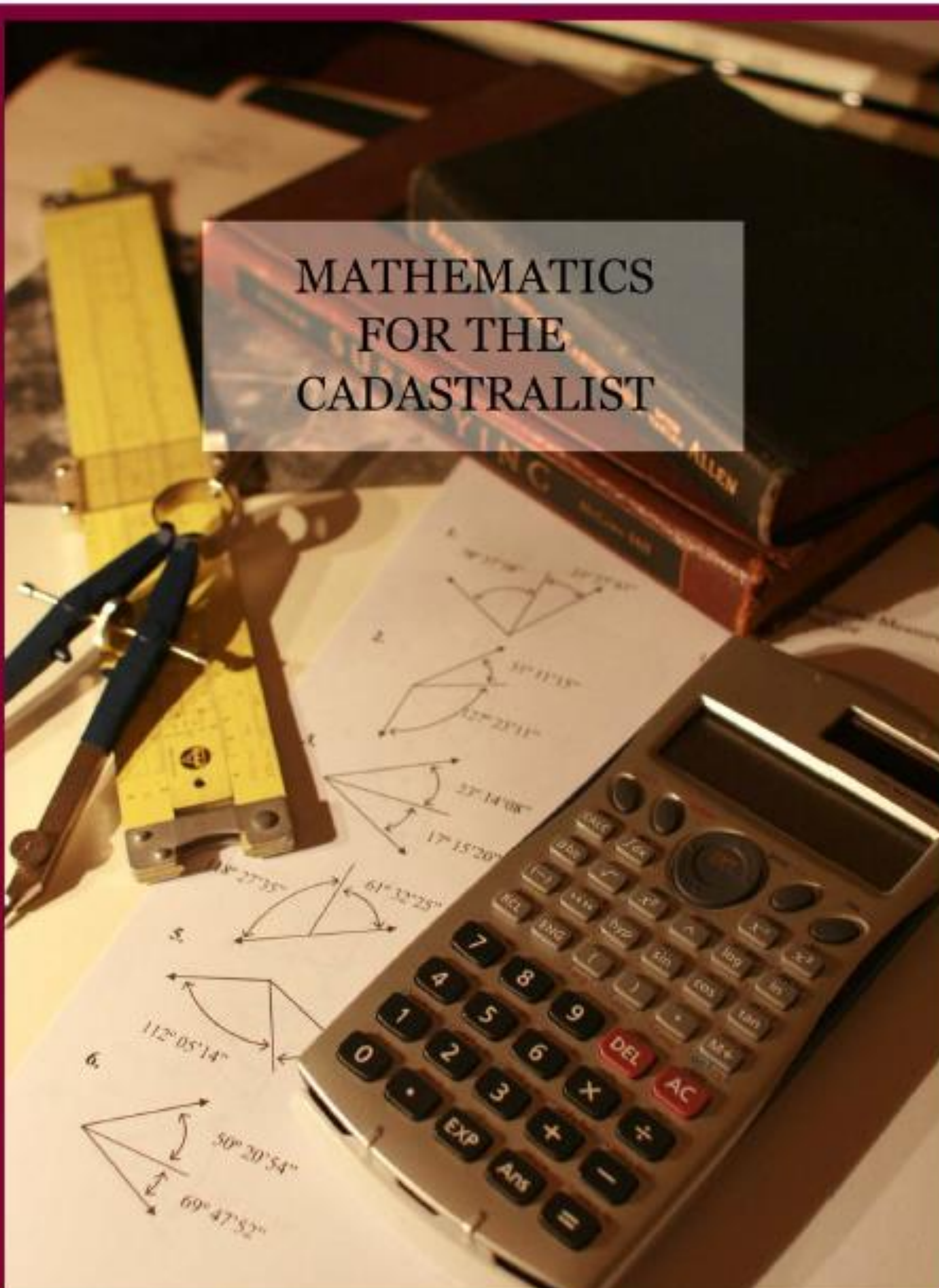
MATHEMATICS FOR THE CADASTRALIST



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FLORIDA ASSOCIATION OF CADASTRAL MAPPERS

In conjunction with

THE FLORIDA DEPARTMENT OF REVENUE

Proudly Presents

COURSE 1

MATHEMATICS FOR THE CADASTRALIST

Objective:

Upon completion of this course, the student will:

Compute triangles.
Perform area calculations.
Understand and use coordinates for boundary calculations.
Understand and use practical applications of coordinates and area calculations in cadastral mapping.

Also, they will be able to:

Use some formula manipulations.
Know some of the basic principles of algebra, geometry & trigonometry.
Use some practical applications of the above in cadastral mapping.

COURSE OUTLINE

DAY ONE

MATHEMATICS FOR THE CADASTRALIST

COURSE OUTLINE

DAY TWO

MATHEMATICS FOR THE CADASTRALIST

COURSE OUTLINE

DAY THREE

MATHEMATICS FOR THE CADASTRALIST

Course Outline

DAY FOUR

MATHEMATICS FOR THE CADASTRALIST

REVIEW

INTRODUCTION AND BACKGROUND INFORMATION

We utilize mathematics in a variety of different ways. In everyday life, we are constantly adding, subtracting, multiplying or dividing. In mapping, we use algebraic, geometric and trigonometric functions to create a graphic representation that we have come to know as a cadastral map. After all, a cadastral map is a mathematically determined representation of a portion of the earth's surface symmetrically plotted to scale upon a plane surface. In this course, we will learn the basics of cadastral mapping as it relates to mathematics. We will explore the basic rules of algebra, geometry, and trigonometry then apply those principles.

Mathematics is an ancient science that deals with logical reasoning and quantitative calculation using numbers, shapes, and various ways of counting and measuring. Modern mathematics has evolved from a simple science to a very abstract field of theory. It is the language used by all the other sciences, and is the basis for precision in many other branches of science.

From the earliest of times, man has used mathematics to define property lines. The Egyptian pyramids reveal evidence of a fundamental knowledge of surveying and geometry as early as 2900 B.C. The Babylonians also had a fundamental knowledge of algebra, were particularly strong in number theory and had developed accurate area formulas for triangles and trapezoids at approximately the same time. Since they used a crude approximation of three for the value of pi ($\boldsymbol{\pi}$), they achieved only rough estimates for the areas of circles.

Neither Egyptian nor Babylonian mathematics dealt with abstract entities or the idea of a mathematical proof. The Greeks developed the idea of the mathematical proof. Pythagoras, a Greek, provided one of the first proofs in mathematics and discovered irrational numbers. The famous Pythagorean Theorem relates to the sides of a right triangle with their corresponding squares.

Ancient knowledge of the sciences was often wrong and wholly unsatisfactory by modern standards. However, the mathematics of Euclid, Apollonius of Perga and Archimedes remain as valid today as they were more than 2,000 years ago. Euclid's "Elements of Geometry" used logic and deductive reasoning to set up axioms, postulates, and a collection of theorems related to plane and solid geometry, as well as a theory of proportions used to resolve the difficulty of irrational numbers. Apollonius, best known for his work on conic sections, coined the term parabola, hyperbola, and ellipse. Another great figure was Ptolemy, who contributed to the development of trigonometry and mathematical astronomy. Still in existence are medieval copies of an ancient geography text attributed to Ptolemy. Ptolemy's text contains maps drawn on a type of conic projection and information on locations and the effects of latitude.

During the Renaissance, improvements in practical astronomy and the development of trigonometry brought better surveying methods and the mathematical tools for creating new and better maps. Although far superior to earlier maps, the maps of the Renaissance left much to be desired.

Today, with all of the advancements in technology, it is possible to have very high quality maps.

BASIC PRINCIPLES

ALGEBRA

Algebra has often been described as "arithmetic with letters." Unlike arithmetic, which deals with specific numbers, algebra introduces variables that greatly extend the generality and scope of arithmetic. The algebra taught in high schools involves techniques for solving relatively simple equations. Algebra is used in the calculation of compound interest, in the solution of distance-rate-time problems, or in any situation in the sciences where the determination of unknown quantities from a body of known data is required.

Algebra is a method of using abstract symbols to represent values so that a formula can be derived for a set of similar problems with different values. In order to accomplish this, there are a set of rules that must be followed. These rules allow for the manipulation of symbols that will isolate a desired unknown and the substitution of values for symbols to solve for that unknown.

Some of the basic rules are as follows:

Commutative Property

An easy way to think of this law is by the word of the property, commutative. The symbols can "commute" to different positions without changing the value of the expression. You will note that this is shown for addition and multiplication only and not for division.

$$a+b = b+a \quad \text{or} \quad ab = ba$$

Associative Property

The associative property allows the association of certain symbols with different symbols without changing the value of the expression. Here again, this is true only for addition and multiplication.

$$a+(b+c) = (a+b)+c \quad \text{or} \quad a(bc) = (ab)c$$

Distributive Property

The distributive property tells us that $a(b+c) = ab+ac$ or that a is distributed over the expression (b+c) without changing the expression a(bc).

Identity Property

The identity property is separated into the additive and multiplicative identity property. This property simply states is that multiplication by 1 and addition of a 0 does not change the value of an expression.

Additive: a+0 = a
Multiplicative: 1a = a or a1 = a

Inverse Property

For every real number **a**, there is a single real number **–a** such that a+(-a) = 0 and (-a)+a = 0. For every real number a, there is a real number 1/a such that a(1/a) = a and (1/a)a = 1.

Formula Manipulations

The foregoing rules will allow the orderly manipulation of abstract symbols and the ultimate solving of a formula for an unknown value. If the formula has parentheses and/or brackets, start inside of the brackets and combine all terms. Work from there to the rest of the formula.

If the formula has no brackets or parentheses, start by simplifying all powers and roots starting from the left to the right. Next, perform all multiplications and divisions starting from the left and working right. Finally, perform all addition and subtractions again working from the left to the right.

A simple and easy rule to remember is: when an expression contains and equal sign, then whatever is done to one side of the equal sign can be done to the other side without changing the value of the expression.

This is a very abbreviated review of some of the basics of algebra. These rules will be used in the following as we proceed in the course.

GEOMETRY

The word geometry is derived from the Greek meaning "earth measurement." Although geometry originated for practical purposes in ancient Egypt and Babylonia, the Greeks investigated it in a more systematic and general way. The Romans followed with many pragmatic applications of the Greek theories. The problems of displaying the spheroid shape of the earth on a two dimensional surface (paper) has plagued mappers through the ages. The problem has been approached both through graphic and mathematic solutions. Most of these problems have been worked out for the cadastral mapper. For the purpose of this class, only the "plane" or two dimensional surface will be treated. The primitive operating units of geometry are points, lines and polygons (any multi-sided figure).

The Primitives

The basic units of points, lines and polygons are used in mapping fields to model reality. A simple description of each follows:

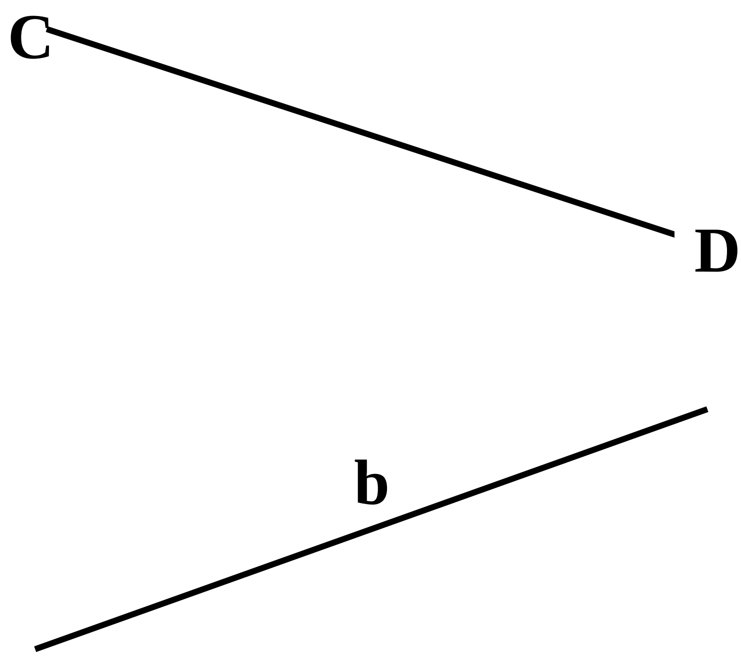
Points

A point is an element having position, but not dimensions as in the intersection of two lines. They can stand on their own, such as the location of all power poles or they can be definite position as in a scale. Lines are often referred to as an infinitesimal number of points.

Lines

A line is a straight set of points that extends off to infinity in two directions. A line can have infinite length, but zero width and zero thickness. It is named by any two points on the line, or it can also be named by one small letter (a, b or c).

Lines are an endless set of points and connect points. They come in many forms. For this discussion, lines will be restricted to “straight” or “curved.” By “straight,” it is meant that the line is constant in direction. When the term “curve” is used, it shall apply to a regular circular curve (a constant distance from a unique point). Lines in this discussion will also be finite. A vector is a line with a specified direction and distance.



Polygons

A polygon is a closed plan figure having three or more (usually) straight lines. Some frequently used polygons are the square, the rectangle, the parallelogram, the trapezoid, and triangle. There are many other regular and irregular polygons with any number of sides and angles.

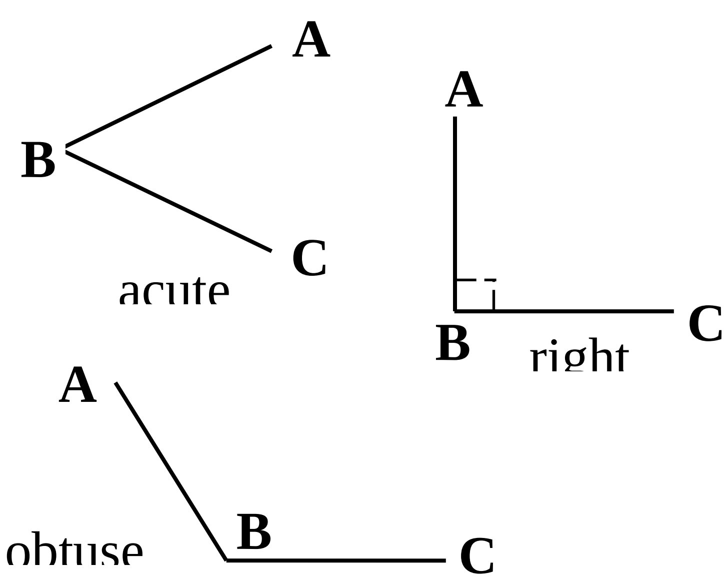
The Rules

Some of the rules for using these plane geometric units need to be reviewed at this point. As with all mathematics, without a progressive understanding of the foregoing rules, the subsequent rules have little meaning or value. Points will be discussed as a special topic area later (coordinates). Lines and polygons have a few basics that all mappers should be familiar with.

Lines intersect as points and these points are sometimes referred to as vertices. When two lines cross they are said to intersect at an **angle**.

The Angle

An angle is the union of two lines with a common endpoint. That point is called the vertex with the lines being called the sides of the angle. **An angle is also a measure of rotation.** One complete rotation is measured as 360º (*in the United States, angles in a unit circle are said to equal 360 degrees).* The system for subdividing a degree is known as sexagesimal. This is a system based upon the number SIXTY. There are sixty minutes in a degree, and sixty seconds in a minute. The symbol for a minute is the same as the foot symbol (**'**) and the symbol for a second is the same as the inch symbol (**"**). If the lines point in the same direction, then the angle between them is zero. In geometry, an angle is measured in degrees from 0º to 180º. **The number of degrees indicates the size of the angle.** The opposite angles of this intersection are equal. This concept of angles and lines is very important to the discussions later.



When two lines are a constant distance apart, they are referred to as parallel. This is true of straight or curved lines. When a third straight line intersects a pair of parallel straight lines, the angles at which the third line intersects the other are equal.

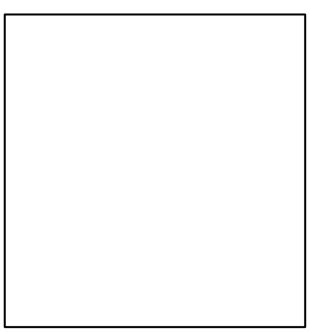
When three or more lines are placed such that they form a closed figure, they are called a polygon. A polygon has a special name to cadastral mappers: a parcel. Polygons have many definable features. They have area which is defined in some square units. The sum of the interior angles is equal to the number of angles minus two times 180 degrees.

This rule is used by surveyors to check angular closure of angles turned in the field with the angles that the closed polygon should have. This is true even if the polygon contains a curve. In that case, the angles to chord are used.

Just as the angular closure can be checked so can the mathematical closure of a polygon. The area of a polygon can be derived from this mathematical closure routine with just a few more computations. The common polygons of geometry are the square, rectangle, parallelogram, and triangle. With the exception of the triangle, these are four-sided figures or quadrilaterals. Each has a set of formulas associated with it, in regard to area. The figures and the formulas are shown below:

A = area **P = perimeter**

The Square



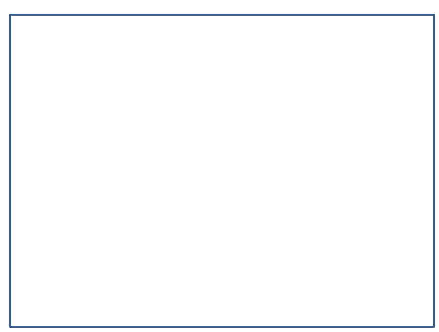
a=b=c=d

A = ab or (axb) = bc = cd = ad

P = a + b + c + d

All angles are 90 degrees

The Rectangle



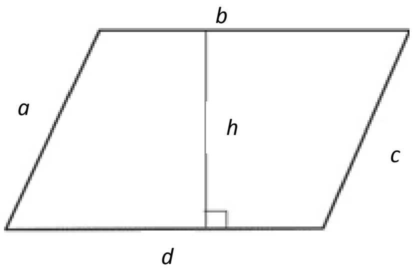
a=c and b=d

A = ab = cd

P = a + b + c + d

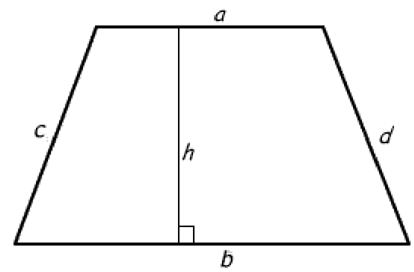
All angles are 90 degrees

The Parallelogram



a=c and b=d
sides a and c are parallel and
sides b and d are parallel
h=perpendicular distance between the parallel sides
A = hb = hd
P = a + b + c + d

The Trapezoid



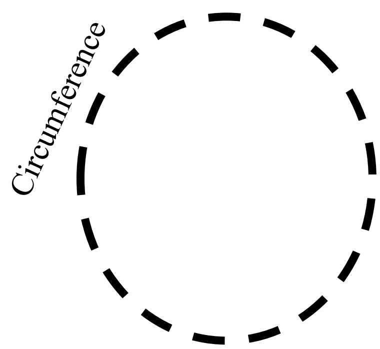
no sides equal

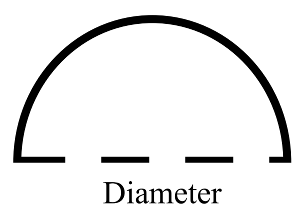
sides b and d are parallel

A = h(b+d)/2
P = a + b + c + d

The Circle

The circle is a polygon without "straight lines". A circle is a set of points that are all a fixed distance from a given point. The given point is known as the center. The distance from the center to a point on the circle is called the radius. The circumference is the distance you would have to walk if you walked all the way around the circle. The diameter is the farthest distance across the circle; it is equal to twice the radius. In descriptions, we deal most often with a part of a circle, known as the arc.

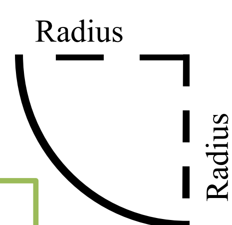




Some Applicable Formulas:

A=piR² **C**=2piR **pi**=3.1416…

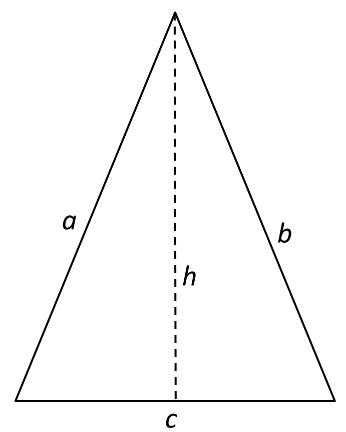
__A__=area; __C__=circumference; __R__=radius; & __pi__=the number of times a radius can be wrapped around the circumference.



CIRCLE	=	**360°**	*(360 PLUS DEGREES SYMBOL)*
MINUTE	=	**60'**	*(60 PLUS MINUTE SYMBOL)*
SECOND	=	**60"**	*(60 PLUS SECOND SYMBOL)*

A special polygon that has some very unique features is used in these computations. This polygon has such a special place in mathematical study that it claims an area all to itself. The polygon is a triangle and the study of it is known as trigonometry. The shape and basic formulas are listed below.

The Triangle



A = (hc)/2

P = a + b + c

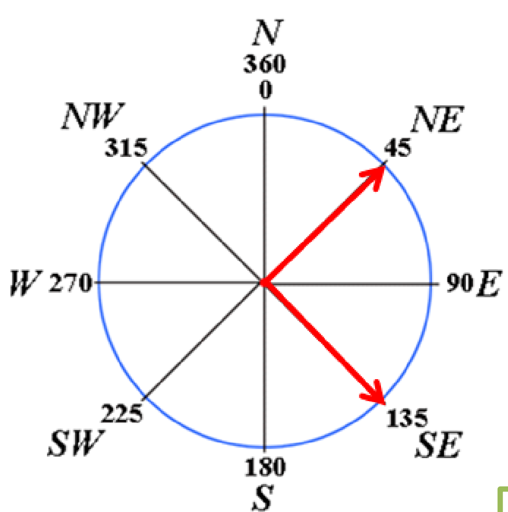
A large part of the course will be spent working with triangles and formulas. Before we discuss trigonometry, we will cover Angular Measure and its practical application.

ANGLES AND BEARINGS

BEARING AND DISTANCE RELATIONSHIP

The legal description is a narrative that attempts to describe a parcel or real property. There are many forms of legal descriptions but for now we will concentrate on the metes and bounds descriptions. In particular, we shall be addressing the bearing and distance type of metes and bounds legal description. To do this, we must establish a set of basic rules and cite some standards. The name of an object that has both direction and distance in mathematical terms is a vector. The direction can be defined in two ways in a normal description.

The first method is by **azimuths**. This method is related to the navigation systems used in the past and in some ways allows ease of computations. The direction is based upon 360 degrees and is measured from either the south or the north. The direction of the origin must be specified or known before this system can be used. A hand compass normally has an orientation of north for azimuthal orientation.



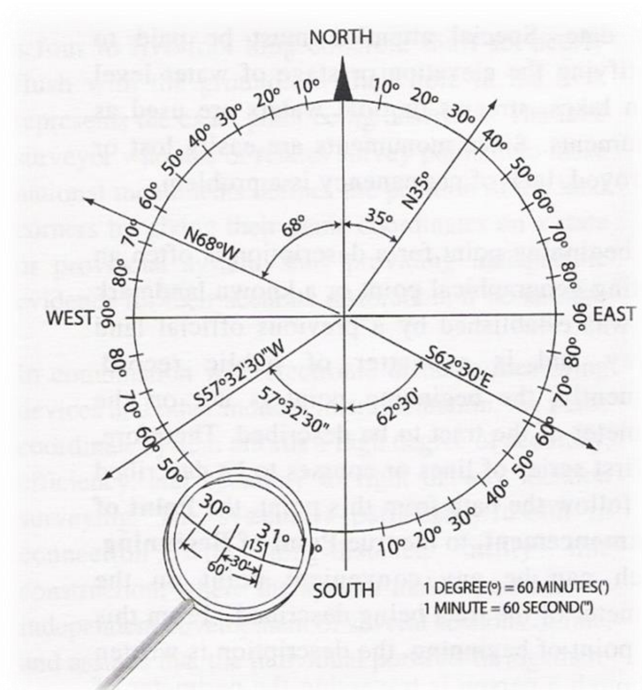
Example: The azimuthal direction 45° is called northeast (NE), and 135° is called southeast (SE).

The azimuth of a line is defined as the clockwise angle from the north end or south end of the reference meridian.

Azimuths are most commonly measured from the north end of the meridian.

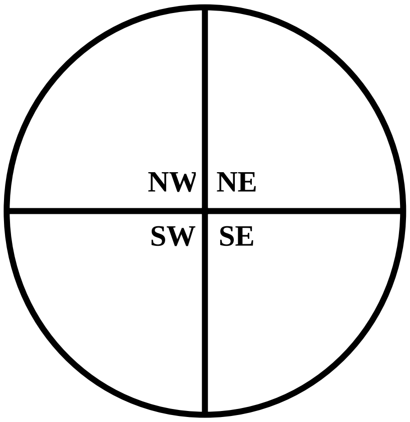
The other more commonly used system for expressing a direction is **the bearing**. The bearing system divides the 360 degree circle into four quadrants of ninety degrees each.

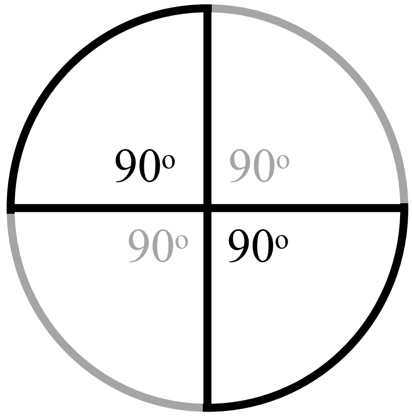
Bearings are best explained in reference to the four quadrants of a surveyor compass.



The zero for the north quadrants is north and the zero for the south quadrants is south. Ninety degrees is at east and west. The method is to specify either south or north, the angle from the origin and the east or west direction. The maximum angle for any bearing is ninety degrees normally specified as East or West. The minimum angle for this system is zero degrees and is normally expressed as North or South.

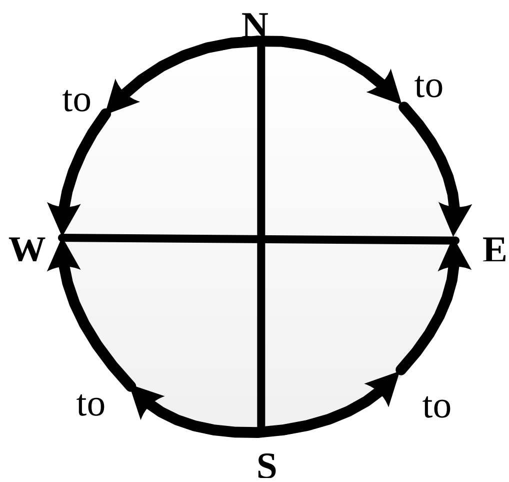
Surveyor's Compass. These are called the Northeast, Northwest, Southwest and Southeast quadrants. **<u>The measurement of a bearing does not exceed 90 degrees.</u>**





Surveyor's Quadrants Direction

1. North to East
2. South to East
3. South to West
4. North to West



Remember – measurement of a bearing does not exceed 90 degrees in a surveyor's quadrant & will always be **from** North or South **to** East or West!

Bearings can be very useful in computing the mathematical closure of a parcel of property. The derivation of angles from bearings and the computation of bearings from angles *(given a basis of bearings)* is no problem if the basic, foregoing definitions are constantly referred to. *(In computations of bearings, either from angles or from bearings, be very liberal in the use of sketches for a more complete understanding.)* **"A picture is worth a thousand words."** It must be kept in mind that the system for angles is the sexagesimal system and not a decimal system. The use of sixty as a base is not hard as long as it is remembered. The other part of this vector system is the distance.

The distance is normally measured in feet to the nearest one hundredth of a foot. This measurement is the present day convention. The past is varied, but the chain was most common. Many of you have dealt with Spanish land grants where varas were used or you may be familiar with the variations of the chain such as poles or rods. Unlike the vara, the chain has a constant value within the U. S. survey system *(although the length may vary as the individual chain was worn from use).* Calibration of the chain is and was very important to consistent measurement of properties. The U. S. Government maintains a standard of measurement *(for many different forms of measurement)* by which daily measuring devices may be compared for accuracy.

> The difference between accuracy and precision is very important. The accuracy is how well a measurement compares to a standard and precision is the degree to which the measurement is made. When accuracy and precision are the same, the measurement is valuable. An example of how this is not met may be that a survey may be accurate to one foot, but the precision to which it was measured was 00.01.
>
> Another example is that a property is measured to the nearest one hundredth of a foot with a chain that was manufactured 101.00 feet long. If this chain is thought to be 100 feet in length, then the survey can be very precise, but not very accurate.

The point that is important to remember is that if the chain or distant measuring device is not calibrated to a standard, then the distance is suspect at best. The distance is important as the second part of the "vector". These two items are parts of a system for describing a position or a set of positions in two dimensional space. This is a "polar" coordinate system. We will later discuss the "rectangular" coordinate system.

To summarize the metes and bounds call, it is important to remember that it consists of two parts: bearing and distance. Sometimes, these are not described precisely, as in so many feet "more or less" or "in a southwesterly direction." Either is perfectly legal (and may even be better than a more specific distance or direction), but still hard to plot. For imprecise calls such as these, we may need to apply our knowledge of mathematics, geometry and trigonometry to arrive at the correct position and configuration of the property.

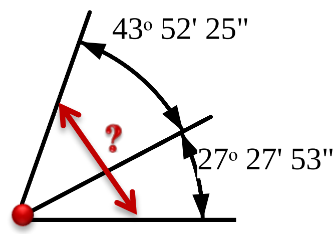
ANGULAR MEASURE

The Addition of Angular Measure

Write each angle down as degree - minute - second. Add as in regular addition, but in **vertical columns,** moving from right to left as shown below. Since any part of a degree (minutes or seconds) CANNOT be over 60, subtract 60 (or a multiple of 60) from whatever column needs it, then add one (or more) to the column to the left. **Work in columns**.

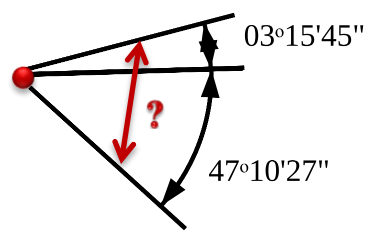
EXAMPLE 1:

	degrees	**minutes**	**seconds**
	43º	52’	25”
+	27º	27'	53"
	70º	79’	78”
-	00°	00’	60”
	70°	79’	18”
+	00°	01’	00”
	70°	80’	18”
-	00º	60’	00”
	70º	20’	18”
+	01º	00’	00”
	71º	**20’**	**18”**



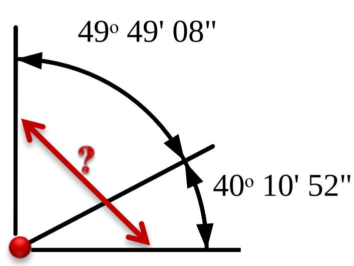
EXAMPLE 2:

	degrees	**minutes**	**seconds**
	47º	10’	27”
+	03º	15'	45"
	50º	25’	72”
-	00°	00’	60”
	50°	25’	12”
+	00°	01’	00”
	50°	**26’**	**12”**



EXAMPLE 3:

	degrees	**minutes**	**seconds**
	49º	49’	08”
+	40º	10'	52"
	89º	59’	60”
-	00°	00’	60”
	89°	59’	00”
+	00°	01’	00”
	89°	60’	00”
-	00º	60’	00”
	89º	00’	00”
+	01º	00’	00”
	90º	**00’**	**00”**



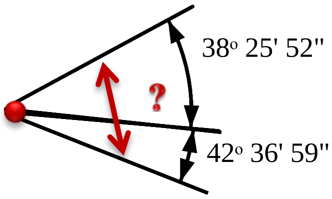
Notice that ALL the angle parts (columns) are at least two numbers. Keep each column two numbers, even when done with the column. It just doesn’t look correct to show an angle or bearing as 00°01’0”. Did a number get left out? Will the reviewer know what is happening?

The Addition of Angular Measure

Problems:

Working Area:

1.



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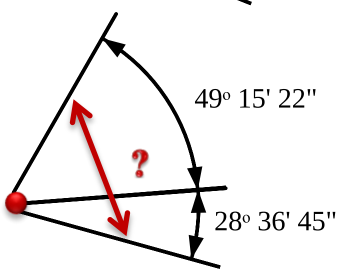

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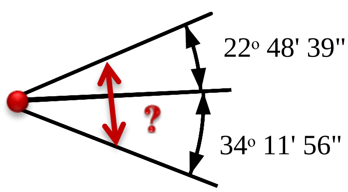

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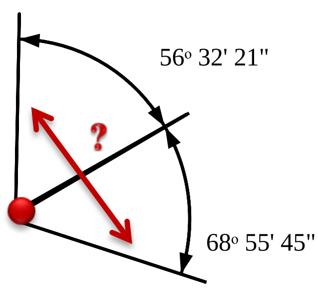

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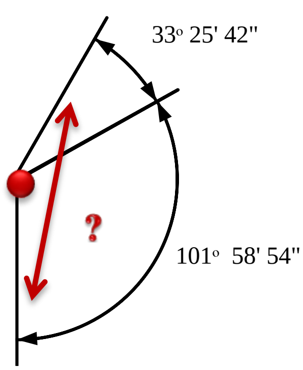
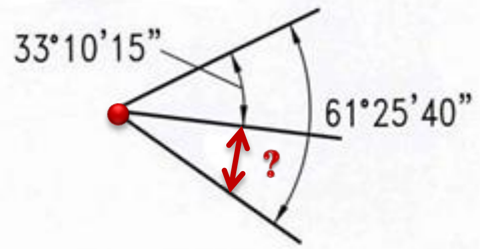


The Subtraction of Angular Measure

Pick the angle with the larger number and then subtract the smaller one. Do it in **columns** (just as in addition on the previous pages). Remember minutes or seconds (in columns) cannot be larger than **59** or smaller than **00**. To borrow, you **REDUCE** the number to the **left** and **INCREASE** one to the **right**. That is, you borrow ONE from the left and add **60** to the right figure.

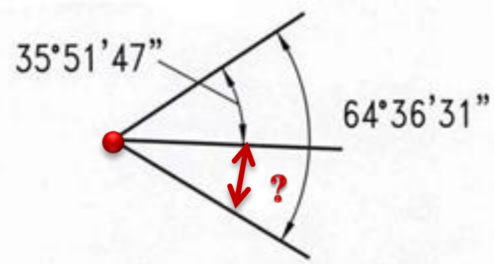
EXAMPLE 1:

	degrees	**minutes**	**seconds**
	61º	25’	40”
-	33º	10'	15"
	28º	**15’**	**25”**



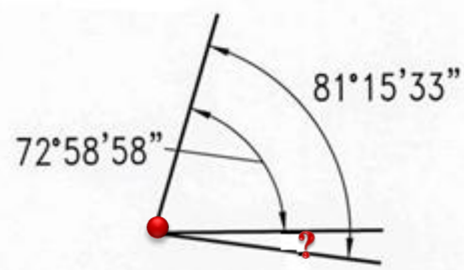
EXAMPLE 2:

	degrees	**minutes**	**seconds**
	64º	36’	31”
		-01'	+60"
	64º	35’	91”
	-01°	+60’	+00”
	63°	95’	91”
-	35º	51’	47”
	28º	**44’**	**44”**



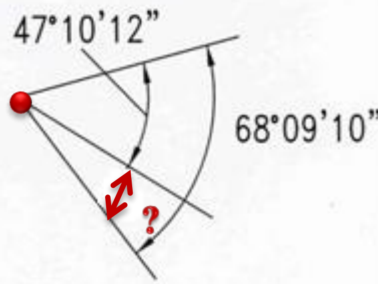
EXAMPLE 3:

	degrees	**minutes**	**seconds**
	81º	15’	33”
		-01'	+60"
	81º	14’	93”
	-01°	+60’	+00”
	80°	74’	93”
-	72º	58’	58”
	08º	**16’**	**35”**



EXAMPLE 4:

	degrees	**minutes**	**seconds**
	68º	09’	10”
		-01'	+60"
	68º	08’	70”
	-01°	+60’	+00”
	67°	68’	70”
-	47º	10’	12”
	20º	**58’**	**58”**

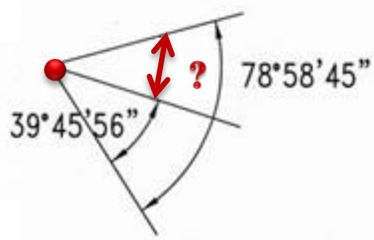


Subtraction of Angular Measure

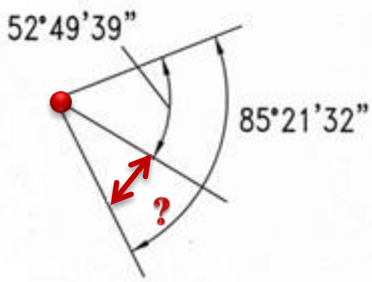
<u>Problems:</u>

<u>Working Area:</u>

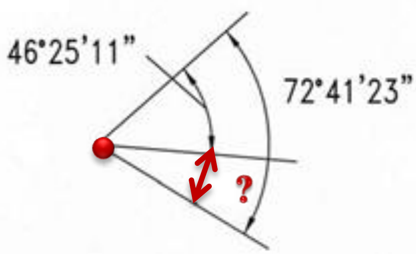
1.



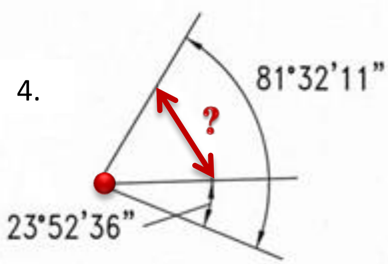
2.



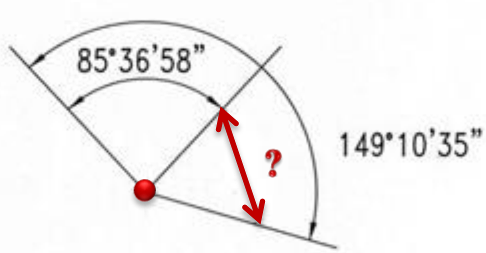
3.



4.



5.



Multiplication of Angular Measure

Mappers do not use multiplication of angular measure very much, but it is done basically the same as regular multiplication. Just **start** on the **right** working your way to the left and stay in a column until you complete it. See the example below.

	degrees	minutes	seconds	
	086º	**038'**	**043"**	→Use hundreds to write down the angle.
*			**5**	→Multiply by 5 times. Start on the right!
	430º	190'	215"	There are 60 seconds in one minute; Therefore, divide 60 into 215". It goes in 3 times (3 * 60 = 180) with 35" remaining.
-	000º	000'	180"	→Subtract the 180 seconds.
	430º	190'	**035"**	The remaining 35". **Done with seconds.**
+	000º	003'	000"	→Add the new 3' to the 190'.
	430º	193'	035"	Add to get the total minutes.
-	000º	180'	000"	→Then reduce the minutes by subtracting 180 (3 times 60), leaving 13' in the minute column. **Done with the minutes.**
	430º	**013'**	035"	
+	003º	000'	000"	→Add the three degrees to the 430° to get the answer.
	433º	013'	035"	

ANSWER: 433º13'35" The resulting answer.

Division of Angular Measure

Surveyors use division of angular measure when they are doing repetitive angle work. That is turning the angle more than once and dividing that total result (multiple additions to the angle by the number of turns of their theodolite). It was done to reduce angle error. Mappers may occasionally use this method. It is the basic division process. **Start** on the **left** and stay in columns till done with that column! Use hundreds in the angle.

	degrees	minutes	seconds
	0**86º**	0**38'**	0**43"**
5/	**433º**	**013'**	**035"**
-	430º	000'	000"
	003º	013'	035"
+	000º	180'	000"
	000º	193'	035"
-	000º	190'	000"
	000º	003'	035"
+	000º	000'	180"
	000º	000'	215"
-	000º	000'	215"
	000º	000'	000"

The answer.
→433°/5 = 86° with 3° remaining.
→Subtract 5*86° leaving 3°.
3° left = 60' * 3° = 180'.
→Add 180' to the 13'. **Done with degrees.**
→193/5 = 38' with 3' remaining.
Subtract the 5*38' leaving 3'.
3' left = 60" * 3' = 180".
Subtract 3' and add to the 35". **Done with minutes.**
→215"/5 = 43" with none left.
Subtract the 215".
None left. **Done with seconds.**

ANSWER: 86º38'43"

Multiplication and Division of Angular Measure

Problems: Working Area:

1. $45^{\circ}\ 35'\ 49''$
   $\underline{*\qquad\qquad 6}$

2. $69^{\circ}\ 56'\ 59''$
   $\underline{*\qquad\qquad 4}$

3. $89^{\circ}\ 31'\ 23''$
   $\underline{*\qquad\qquad 9}$

4. $6\overline{)\,240^{\circ}\ 36'\ 30''}$

5. $5\overline{)\,450^{\circ}\ 55'\ 45''}$

6. $4\overline{)\,360^{\circ}\ 28'\ 36''}$

ANGULAR MEASURE AND DECIMALS

Converting Angular Measurement to Decimal Degrees

(Convert a degree measurement into a decimal degree measurement.)

In today's world, your electronic calculator is the fastest way to do this, but it is not a hard process. First, we will do it manually working the given degree, minutes and seconds measurement **from right to left,** then we will check out your calculator. Start on the right. Don't forget the columns. Remember the 36° is one column, the 15' is another and the 21" is the last column.

EXAMPLE: 36º 15' 21"	21 ÷ 60 = 0.35	(**divide** the seconds)
	0.35 + 15 = 15.35	(**add** result to the
	15.35 ÷ 60 = .2558333	(**divide** the minutes)
	.2558333 + 36 = 36.2558333	(**add** result to the degrees)

ANSWER: 36.2558333 is the decimal equivalent of 36º 15' 21"

~~~~~~

<u>Drill:</u> Find the decimal equivalent for the following.

1. 89º 27' 43" ____
2. 27º 34' 51" ____
3. 35º 53' 58" ____
4. 47º 53' 21" ____
5. 125º 21' 34"____
6. 01º 23' 45"____
7. 32º 32' 32"____
~~~~~~

Decimal of Degrees to Degrees-Minutes-Seconds

(Convert a decimal degree measurement into a degree measurement.)

This is the reverse of the process we just completed on the previous page. It is a valuable lesson in case our calculator is not working. We will work the problem manually working from the **left** column **to the right** column and check with our calculators. Work in columns.

EXAMPLE: 36.2558333	**Subtract** out the whole number 36	36º
	36.2558333 - 36 = .2558333	
	Multiply .2558333 * 60 = 15.35	
	Subtract out the whole number 15	15'
	15.35 – 15 = 0.35	
	Multiply 0.35 * 60 = 21	
	No remainder	21”

ANSWER: 36º 15' 21"

~~~~~~

<u>Drill:</u> Find the Degree-Minute-Seconds for the following

1. 25.2345671____

2. 87.9872325____

3. 34.5678923____

4. 83.0003469____

5. 125.673245____

6. 2.34982345____

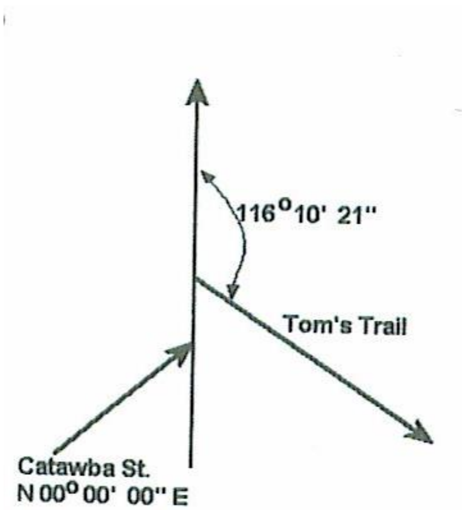
7. 149.1238654____
~~~~~~

USE OF ANGLES AND BEARINGS

CONVERSION OF ANGLES TO BEARINGS

Maps, plats and surveys historically have used bearings and angles in a graphic representation for the Cadastral Mapper. Sometimes, only the angles appear on the survey. Thus, the mapper must compute the bearings from the angles incidental to his work.

1. Find the bearing for Tom's Trail:

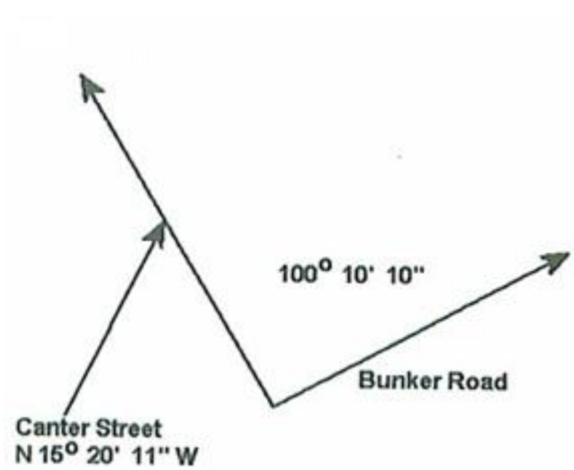


	179º	59'	60"
-	116º	10'	21"
	63º	**49'**	**39"**

Answer: South63°49'39"East

2. Find the bearing for Bunker Road:

	100º	10'	10"
-	15º	00'	11"
	84º	**49'**	**59"**



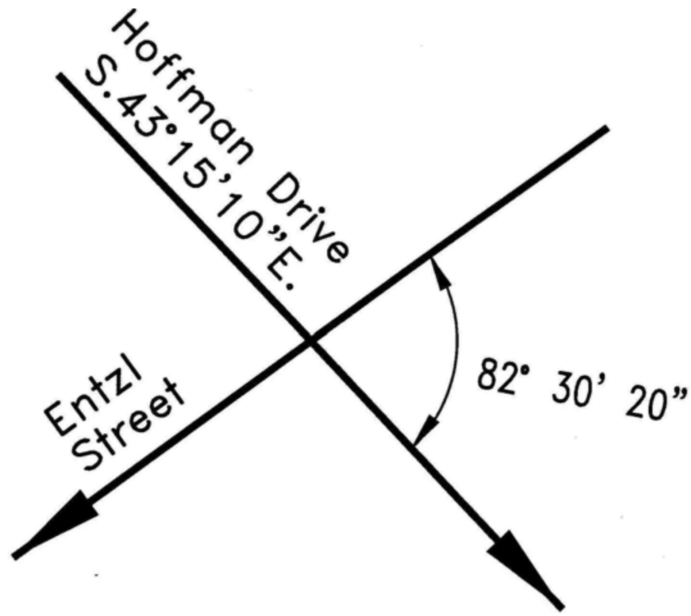
Answer: North84°49'59"East

Seldom will you see a bearing such as N.00° 00'00"E.
Normally, it is written as North.

3. Find the bearing for Entzl Street:

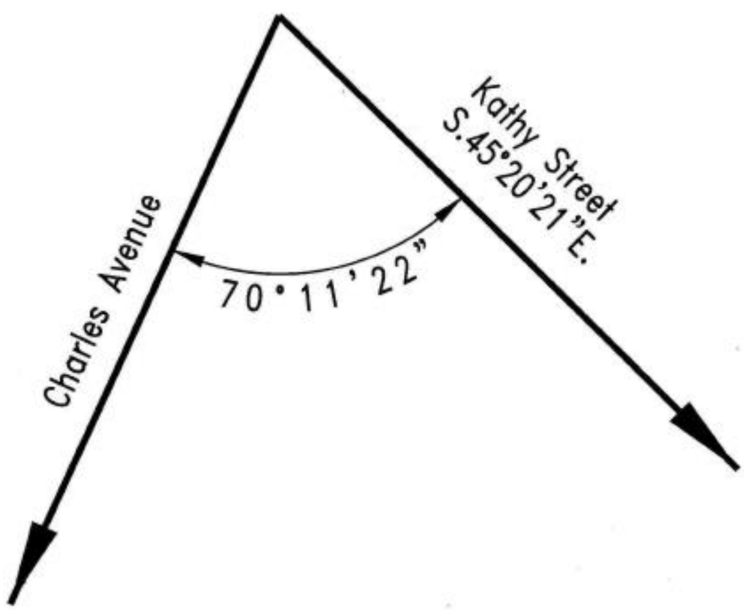
	82º	30'	20"
+	43º	15'	10"
	125º	**45'**	**30"**

	179º	59'	60"
-	125º	45'	30"
	54º	**14'**	**30"**



Answer: South54º14'30"West

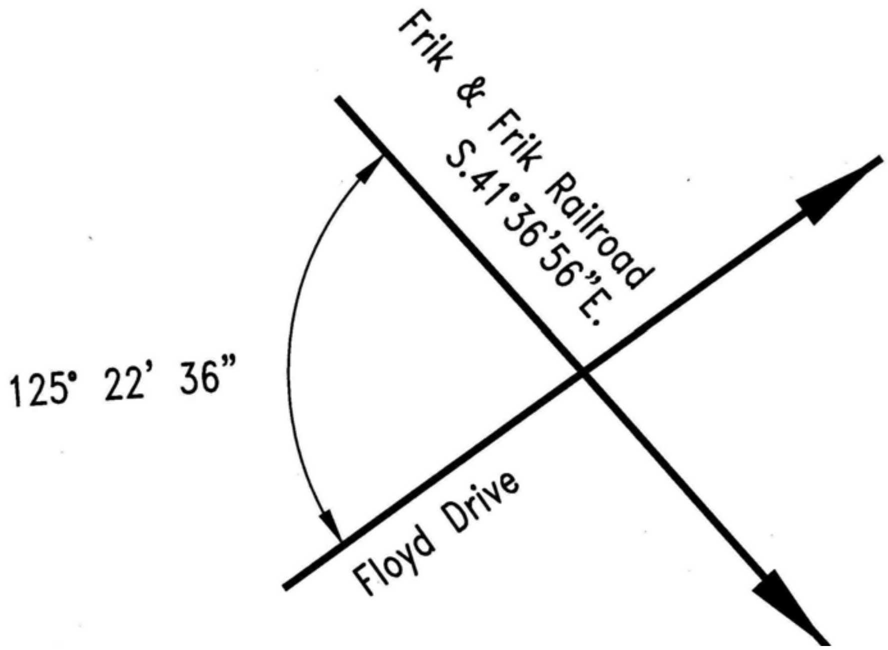
4. Find the bearing for Charles Avenue:



	70º	11'	22"
-	45º	20'	21"
	24º	**51'**	**01"**

Answer: South24º51'01"West

5. Find the bearing for Floyd Drive:



	179º	59'	60"
-	125º	22'	36"
	54º	37'	24"
	54º	37'	24"
-	41º	36'	56"
	13º	**00'**	**28"**

Answer: North13°00'28"East

Use the graphic expectation to check out your answers. Did it look correct? If it looked correct, it may have been!

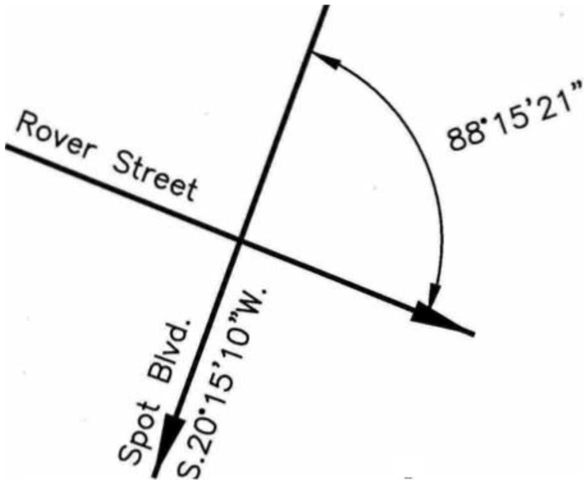
One needs to be very good at drawing the cross over the angle point!

Conversion of Angles to Bearings

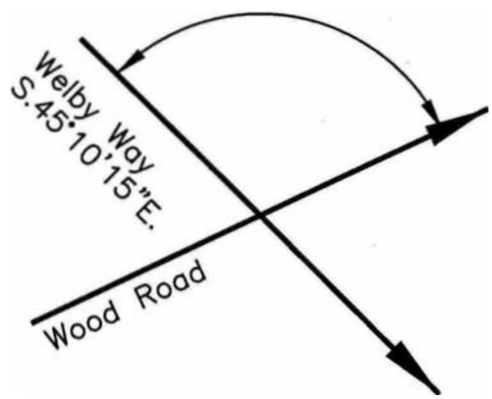
Problems:

Working Area:

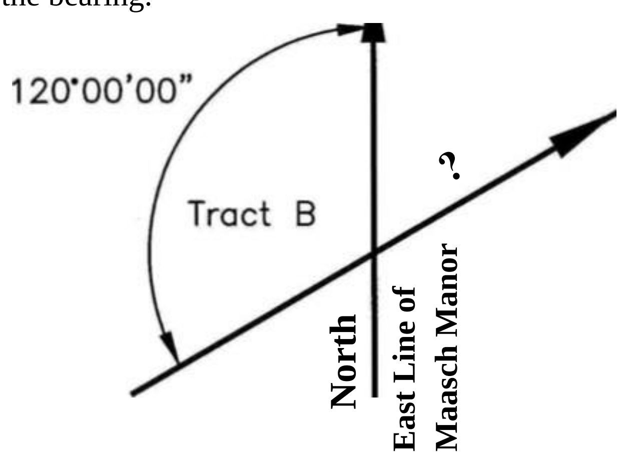
1. **1.** Find the bearing for Rover Street:



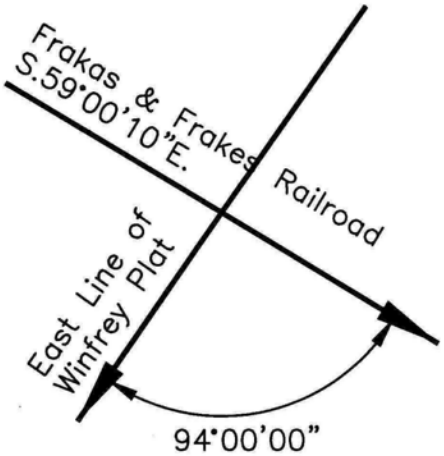
2. Find the bearing for Wood Road:



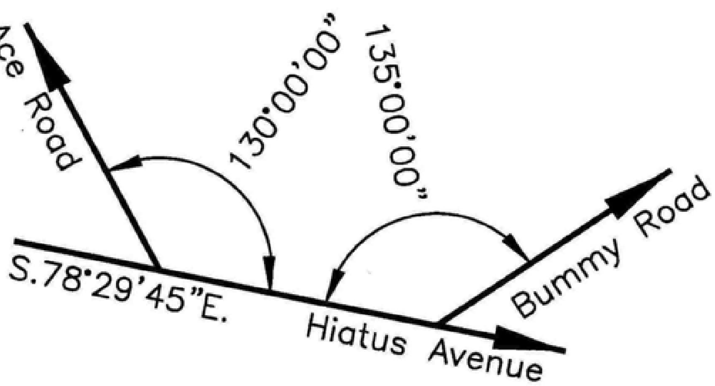
3. Find the bearing:



4. Find the bearing for the East line of Winfrey Plat:



5. Find the bearing for Ace Road and for Bummy Road:

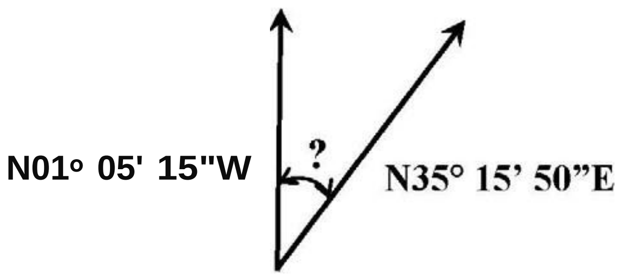


CONVERSION OF BEARINGS TO ANGLES

Most of the time only bearings are found on a survey, plot or map. Therefore the mapper MUST calculate the angle

Examples:

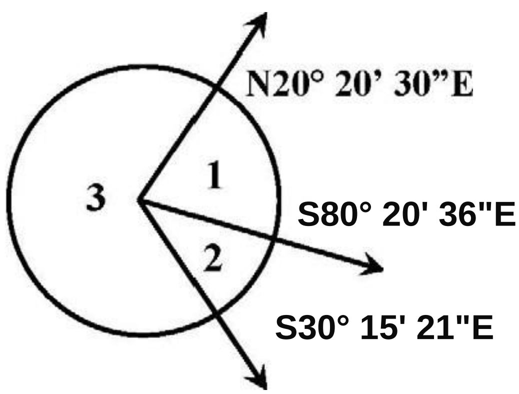
1. Find the angle between the bearings:



	35°	15'	50"
+	01°	05'	15"
	36°	**21'**	**05"**

Answer: 36°21'05"

2. Solve for angles 1, 2 & 3:



Angle 1: Answer: 79°18'54

	89°	59'	60"
-	20°	20'	30"
	69°	39'	30"

	89°	59'	60"
-	80°	20'	36"
	09°	39'	24"

	69°	39'	30"
+	09°	39'	24"
	79°	**18'**	**54"**

Angle 2: Answer: 50°05'15"

	80°	20'	36"
-	30°	15'	21"
	50°	**05'**	**15"**

Angle 3: Answer: 230°35'51"

	50°	05'	15"
+	79°	18'	54"
	129°	24'	09"

	359°	59'	60"
-	129°	24'	09"
	230°	**35'**	**51"**

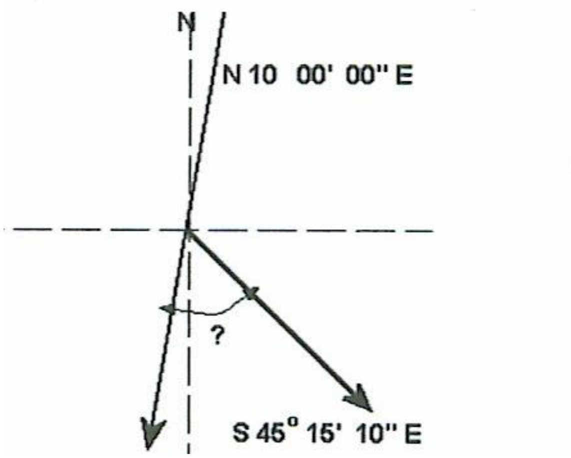
Proof:

	079°	18'	54"
+	050°	05'	15"
+	230°	35'	51"
	360°	**00'**	**00"**

(a full circle)

> Wherever possible, always do a proof of your work!

3. Determine the angle:



	45º	15’	10”
+	10º	00’	00”
	55º	**15’**	**10”**

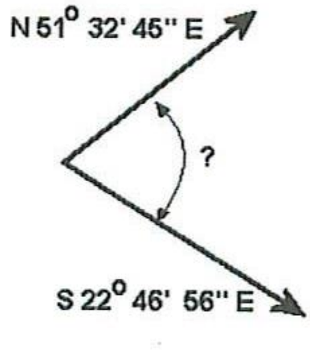
Answer: 55º15’10”

Conversion of Bearings to Angles

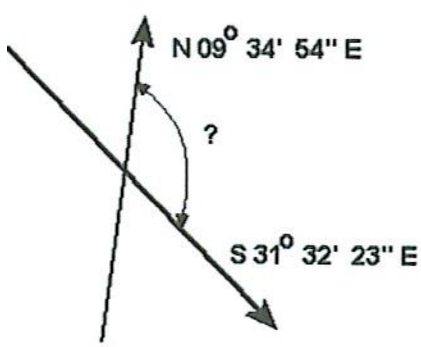
Problems:

Working Area:

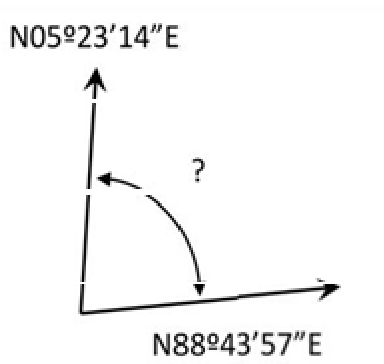
1. Determine the angle:



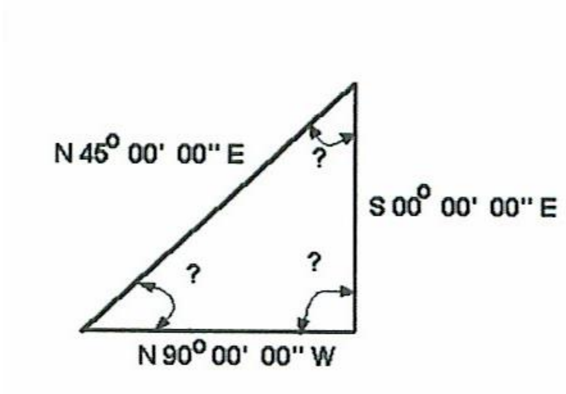
2. Determine the angle:



3. Determine the angle:



4. Determine the interior angles:



While there is a bearing at 90°00’00”, it is seldom used in surveying. Most surveyors convert this bearing into a direction called East or West. By the same token, seldom is 00°00’00” used in a bearing. It is converted to either North or South. However, in this case, assume the surveyor used both.

BEARING WRITING

There are many ways to write a bearing and no set pattern. Most surveyors write them so that no word processing program will split them up and write them on separate lines.

> ***thence N45°45'45"E along the West Line of the said section a distance of 2356.45 feet to the Point of Curvature of a curve to the left having a central angle of 34°34'34" and a radius of 2956.45 feet;***
>
> ***thence, along the arc of the curve, a distance of 345.56 feet to the Point of Tangency;***
>
> ***thence ….***

Wouldn't it be bad to find a bearing cut up by a word processing program that looks like:

> ***S 45° 34'***
>
> ***56" E a distance of …***

Not really sure what you've got, are you?

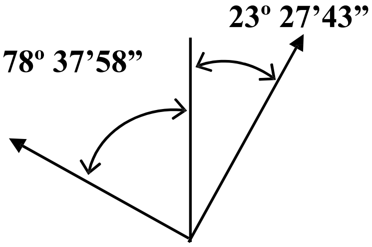
One can write bearings as follows:

a. N 45°45'45" E or (note the space between the direction and the numbers - bad);
b. North 45° 45' 45" East (note the spelling of the direction and a space between the direction and the numbers and spaces between the numbers - bad, bad, bad);
c. N.45°45'45"E. or (note the periods between the direction and the numbers and no spaces between the numbers – good, good, good); or
d. N45°45'45"E (note no space or periods between the direction and the numbers and no spaces between the numbers - good, good, good).

Whichever way you write them is up to you! Most choose one of the last two methods. Also, when writing a description (normally a surveyor's function), the word processing programs will keep the entire bearing together if there are no spaces in it. Just be consistent.

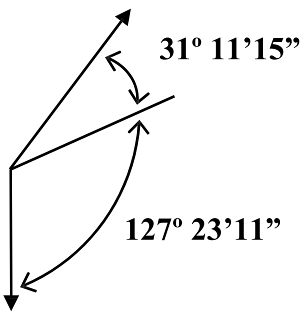
Day 1 Practice
Addition of Angular Measures

1.



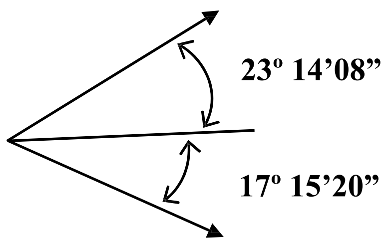
1. ______________________________

2.



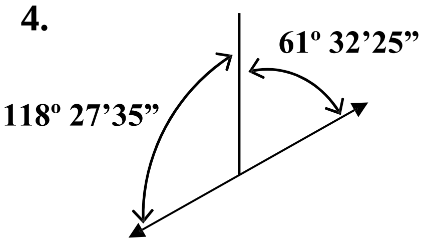
2. ______________________________

3.



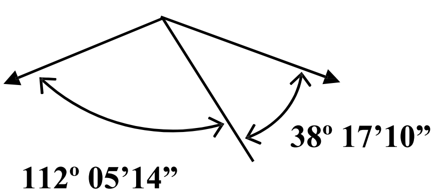
3. ______________________________

4.



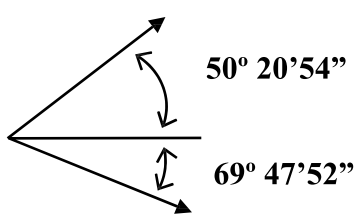
4. ______________________________

5.



5. ______________________________

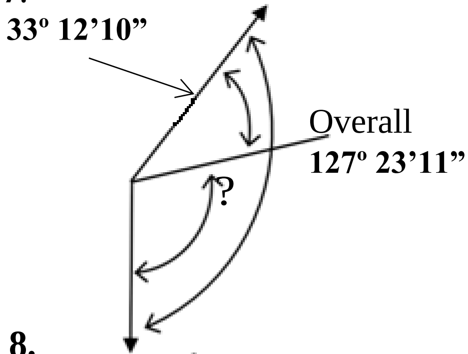
6.



6. ______________________________

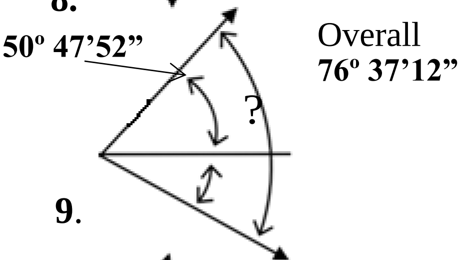
Day 1 Practice
Subtraction of Angular Measures

7.



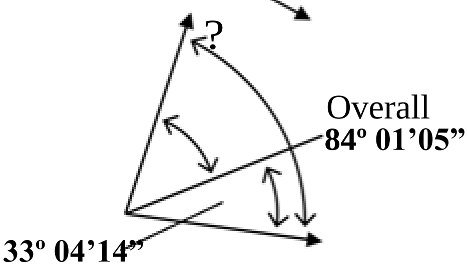
7. ______________________________

8.



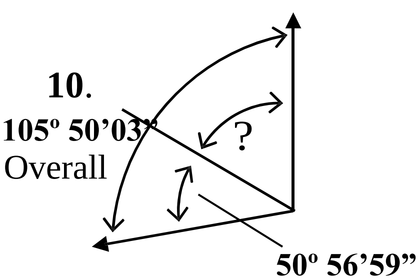
8. ______________________________

9.



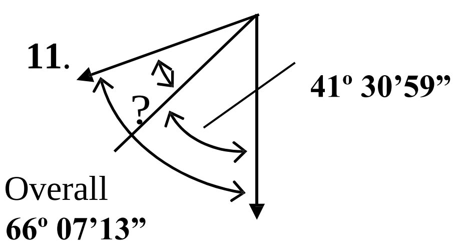
9. ______________________________

10.



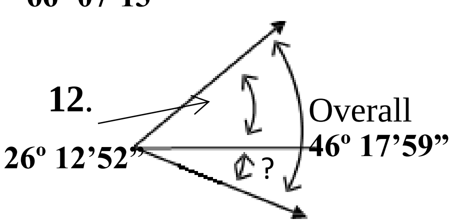
10. ______________________________

11.



11. ______________________________

12.



12. ______________________________

Day 1 Practice
Convert Angular Measurement to Decimal Degree

13. **85º13’46”**______________________________________

14. **89º01’05”** ______________________________________

15. **31º11’15”** ______________________________________

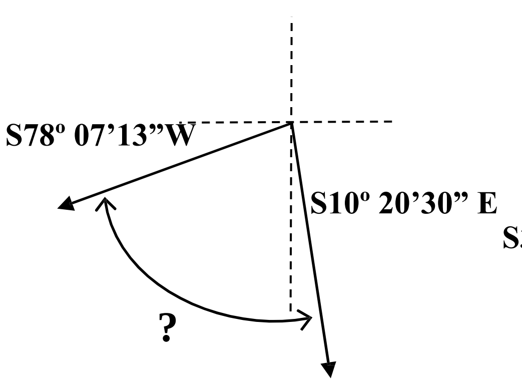
Convert Decimal Degrees to Degrees Minutes Seconds

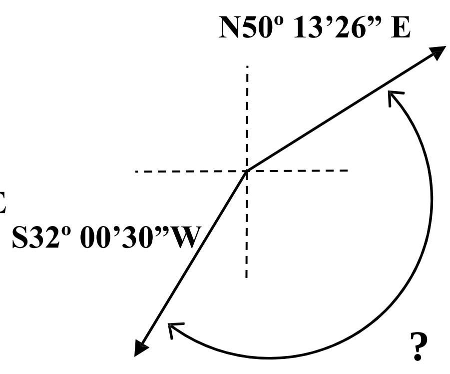
16. **89.39583333**______________________________________

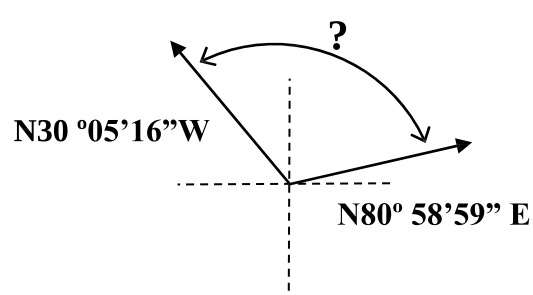
17. **25.61555556**______________________________________

18. **46.38972222**______________________________________

Determine Angles



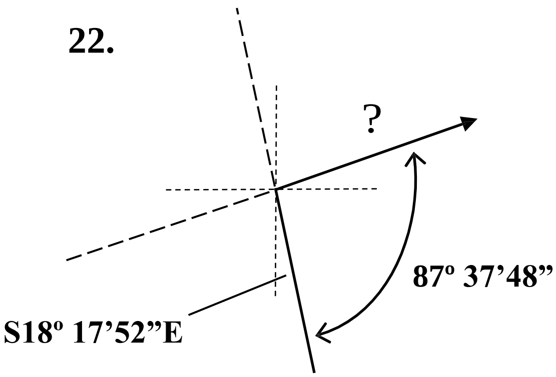




19.__________________________ **20.**__________________________ **21.**________________________

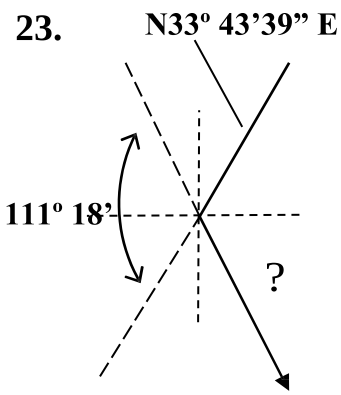
Day 1 Practice
Determine Bearings of Lines with arrows

22.



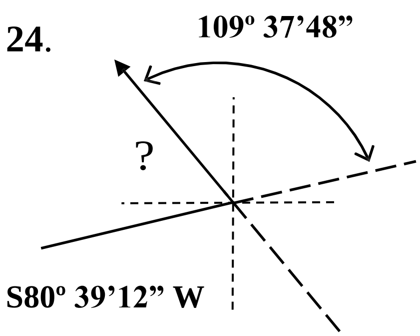
22. ________________________________

23.



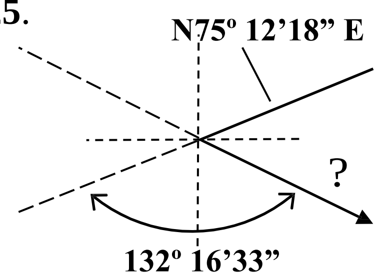
23. ________________________________

24.



24. ________________________________

25.



25. ________________________________

BASIC PRINCIPLES OF TRIGONOMETRY

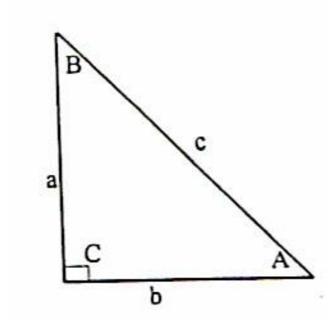
There are basically two triangles that are the basis of the study of trigonometry. These are known as the right triangle and oblique triangle. The right triangle is the key to the whole system of computations of three sided polygons.

Right Triangles

The definition of a plane right triangle is a three sided polygon that has one interior angle equal to 90 degrees with all interior angles equaling a total of 180 degrees. The Egyptians used the plane right triangle to re-establish boundaries of farm land flooded by the Nile for taxing purposes. They also used the plane right triangle to construct the pyramids. They knew that if the sides of a plane right triangle were 3, 4 and 5 or some multiple of these numbers, then the angle opposite the side that was 5 units long (the hypotenuse) was the right, or 90 degree angle. Of course, the plane right triangle can have an infinite number of combinations of different distances and angles.

In order to establish some way to compute the countless plane right triangles, another set of relationships for the plane right triangle had to be developed. At this point it is probably best to establish a typical triangle from which to describe these relationships.

The Right Triangle and Basic Trigonometric Functions



The angles of this triangle are A, B and C and the sides are a, b and c. From this plane right triangle the following formula is stated (without proofs):

$$\mathbf{a^2 + b^2 = c^2}$$

This is known as **Pythagorean Theorem** for the Greek citizen that is credited with the evolution of the formula.

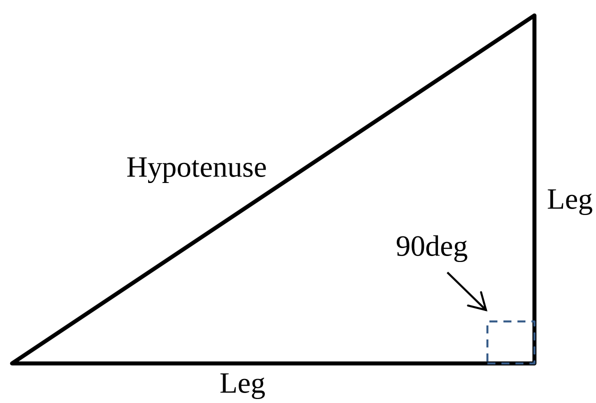
Where c is the hypotenuse, C is the 90 degree angle and the sum of angles A and B equal 90 degrees, the following statements are true (of any right triangle):

Parts of the Right Triangle

The right angle is formed where two lines meet at 90 degrees.

The hypotenuse is the side across from the right angle.

The other two sides are referred to as legs.

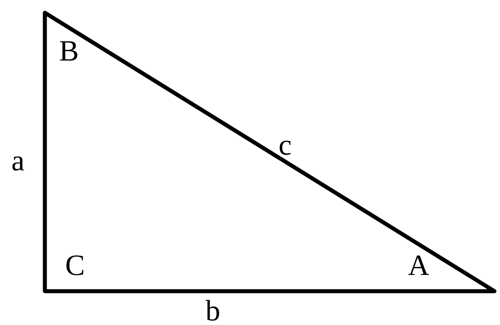


The Right Triangle with All Three Sides

The Pythagorean Theorem

Setting up the triangle:

Normally, a, b & c (lower case letters) refer to the sides of a triangle: a is for altitude, b is for base and c is the third side called the hypotenuse. The hypotenuse is always across from the right angle. Normally, the angles across from the other two sides are labeled with the (upper case letters) A and B. This general rule is applicable to all triangle solving situations and is the standard configuration for all triangles.



The Pythagorean Theorem states:

$\mathbf{a^2 = c^2 - b^2}$ *or*

$\mathbf{a}\ \sqrt{\mathbf{c^2 - b^2}}$

The product of a number multiplied by itself is said to be the square of that number. It is also referred as taking it to the second power **(x^2)**

.

Square Root

What is meant by the square root of a number?

The square root of any number that is a perfect square, is one of the two equal factors of the number. Thus, 3*3 = 9 or 5*5 = 25. The numbers 3 and 5 respectively are the equal factors (square roots) of the nine and twenty-five.

When a number is not a perfect square, the square root cannot be exactly determined. However, with your calculator it can be found approximately to any degree of accuracy that you will need in mapping (example: 25.123456789).

So 3*3 = 9, thus the square root of 9 is 3. The symbol used to indicate the square root sign is the combination of the radical sign and the bar called the vinculum.

Radical sign $\rightarrow \sqrt{\qquad\qquad} \leftarrow$ the bar or vinculum

Examples:

1. $5*5 = 25$ $5^2 = 25$ $\sqrt{25} = 5$

2. $4*4 = 16$ $4^2 = 16$ $\sqrt{16} = 4$

3. $23*23 = 529$ $23^2 = 529$ $\sqrt{529} = 23$

4. $115*115 = 13225$ $115^2 = 13225$ $\sqrt{13225} = 115$

5. $12^2 =$ ____________

6. $2.17^2 =$ ___________

7. $54.21^2 =$ __________

8. $1255^2 =$ ___________

9. $144^2 =$ ____________

10. $23.78^2 =$ ___________

11. $64^2 =$ _____________

12. $5436^2 =$ ___________

13. $\sqrt{3136} =$ ___________

14. $\sqrt{3598609} =$ ___________

15. $\sqrt{3906.25} =$ ___________

Pythagorean example and problem:

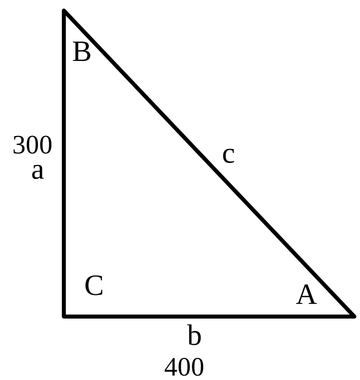
$\mathbf{a^2 + b^2 = c^2}$ can also be written as: $\mathbf{c^2 = a^2 + b^2}$

$c^2 = 300^2 + 400^2$

$c = \sqrt{90000 + 160000}$

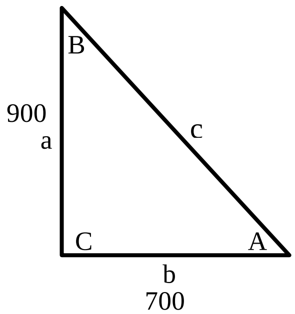
$c = \sqrt{250000}$

$c = 500$



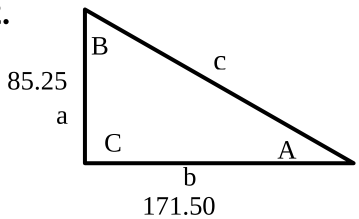
Exercises: Find the hypotenuse (all triangles are right triangles)

1.



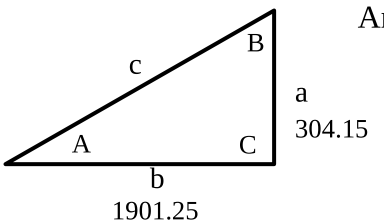
Answer: ________

2.



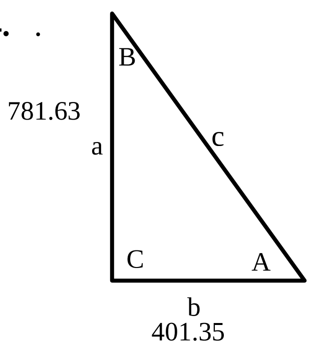
Answer: ________

3. .



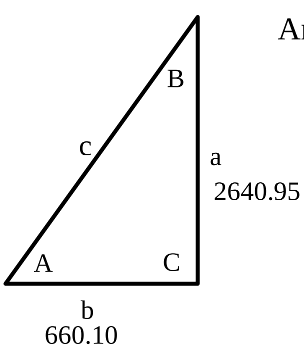
Answer: ________

4. .



Answer: ________

5. .



Answer: ________

Now that we can calculate the hypotenuse, what about the other two sides?

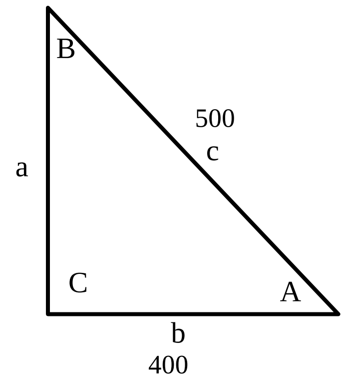
Example:

$a^2 = 500^2 - 400^2$

$a = \sqrt{250000 - 160000}$

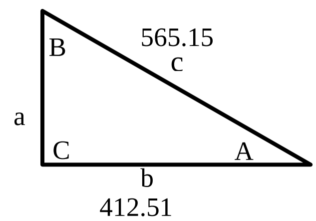
$a = \sqrt{90000}$

$a = 300$

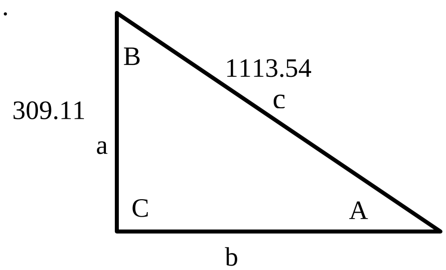


Exercise: Find the missing side. All are right triangles.)

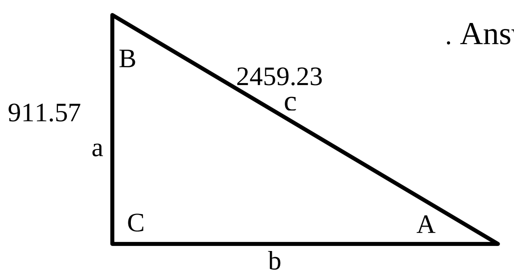
1. . Answer: ________



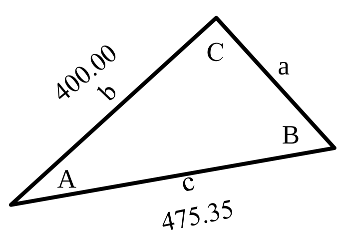
2. . Answer: ________



3. . . Answer: ________



4. . Answer: ________



SINE, COSINE & TANGENT FUNCTIONS

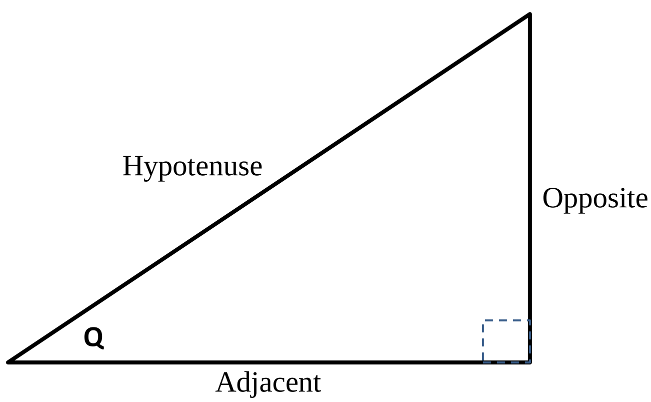
Trigonometry - The study of the properties of triangles and the trigonometric functions and of their applications.

Trigonometric Function(s) - A ratio resulting from the relationship of quotient of the following:

Sine = opposite divided by hypotenuse (full name)

Cosine = adjacent divided by hypotenuse (full name)

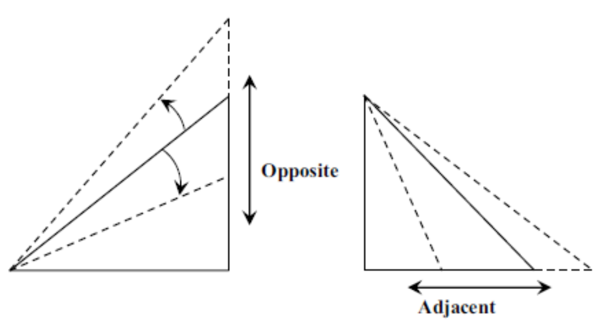
Tangent = opposite divided by adjacent (full name)



While these are different than already discussed, they are correct for trigonometry. What we discussed previously was geometry.

The Ratio

What happens to the opposite side when you increase or decrease the angle Q? What happens when you increase or decrease the adjacent side? What happens to angle Q when you increase or decrease the opposite?



What happens when you have a Sine, Cosine or Tangent?

Years ago (before today's calculators) there were publications of tables that had the relationship between the sides of a triangle and the angles already figured out. The publications were called natural functions of sines, cosines, tangents and cotangents. Today, we have calculators and computers that give us, upon the press of a key, the same numerical values that we may need. A typical old book would be Eight-Place Tables of Trigonometric Functions for Every Second of Arc by Jean Peters which was first published in 1939 and used by surveyors since then.

Without regard as to the application of a triangle solution, let us have a calculator exercise relative to the sines, cosines, and tangent buttons on the calculator.

Given angle:		**(converted to decimal degrees)**	**(get the function)**
sines			
53º 12' 43"	=	decimal 53.211944	sin = .800856231927
22º 15' 21"	=	decimal 22.255833	sin = .378742845476
30º 00' 00"	=	decimal 30.000000	sin = .500000000000
cosines			
01º 12' 30"	=	decimal 1.2083333	cosine = .999777626949
25º 30' 00"	=	decimal 25.500000	cosine = .902585284350
89º 34' 15"	=	decimal 89.570833	cosine = .007490301332
tangents			
44º 23' 45"		decimal 44.395833	tangent = .979129900235
01º 25' 36"		decimal 1.4266666	tangent = .024905178040
88º 23' 55"		decimal 88.398611	tangent = 35.7694872175

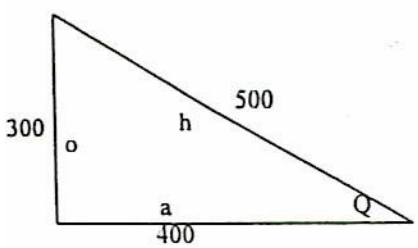
Now convert the (sin, cos and tan) inversely to the degree, minute and second via the decimal equivalent.

Trig value:		decimal equivalent:	angle equivalent:
sin	.99993421	______________________	______________________
	.34478589	______________________	______________________
	.55634892	______________________	______________________
	.75436781	______________________	______________________
	.12345678	______________________	______________________
cos	.99346521	______________________	______________________
	.00034679	______________________	______________________
	.55563289	______________________	______________________
	.21378956	______________________	______________________
	.89722331	______________________	______________________
tan	.98547832	______________________	______________________
	.35245890	______________________	______________________
	.04672981	______________________	______________________
	.87402670	______________________	______________________
	.56487912	______________________	______________________

Basic Functions and the Ratio

The sine (sin) function relative to angle Q:

1)	sin Q = opposite/hypotenuse	sin = o/h
2)	opposite = hypotenuse * sin Q	o = h * sin
3)	hypotenuse = opposite/sin Q	h = o/sin



sin = o/h = 300/500 = .6000

o = h * sin = 500 * .60000 = 300

h = o/sin = 300/.60000 = 500

The cosine (cos) function relative to angle Q:

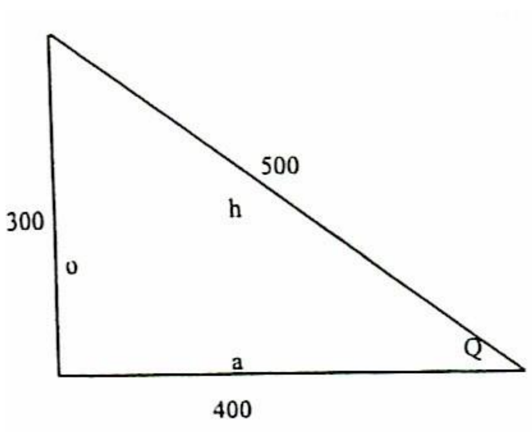
1)	cos Q = adjacent/ hypotenuse	cos = a/h
2)	adjacent = hypotenuse * cos Q	a = h * cos
3)	hypotenuse = adjacent/cos Q	h = a/cos

Examples:

cos = a/h = 400/500 = .8000

a = cos * h = .80000 * 500 = 400

h = a/cos = 400/.80000 = 500



The tangential (tan) function relative to angle Q:

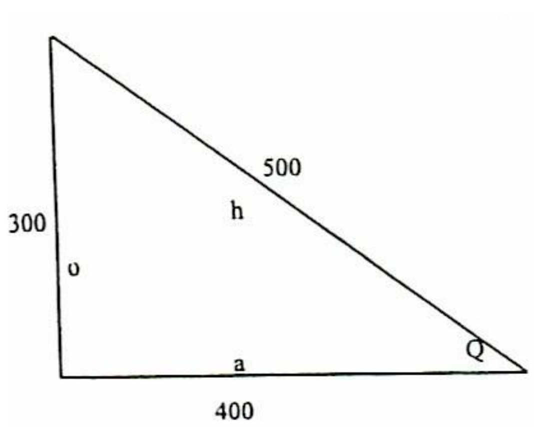
1)	tan Q = opposite/adjacent side	tan = o/a
2)	opposite = adjacent * tan Q	o = a * tan
3)	adjacent = opposite/tan Q	a = o/tan

Examples:

tan = o/a = 300/400 = .750000

o = tan * a = .750000 * 400 = 300

a = o/tan = 300/.750000 = 400



Overall Trigonometry Functions (and how they are figured):

Sine (sin) = opposite / hypotenuse

CoSine (cos) = adjacent / hypotenuse *(It is the previous name, just with "co" placed in front.)*

Tangent (tan) = opposite / adjacent

Cotangent (cotan) = adjacent / opposite *(A previous name with "co" in front.)*

Secant (sec) = hypotenuse / adjacent

Cosecant (csc) = hypotenuse / opposite *(Previous name with "co" in front.)*

External Secant (exsec) = sec Δ – 1 (angle). *These last two are seldom used.*

Versed Sine (vers) = 1- cos Δ (angle)

The Non-Algebraic Solution of the Right Triangle

also

Known as the Method of the Great Indian Chief SOHCAHTOA

There is a non-algebraic solution to right triangle formulas it is called the three-part cover up method. See examples below:

SOH

S = O/H

Sine = Opposite / Hypotenuse

CAH

C = A/H

Cosine = Adjacent / Hypotenuse

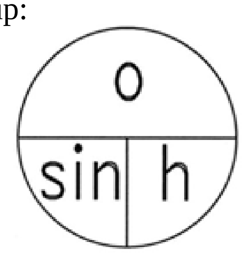
TOA

T = O/A

Tangent = Opposite / Adjacent

<u>JUST REMEMBER THE GREAT INDIAN CHIEF'S NAME</u>: ***SOHCAHTOA***

Whatever function needed is covered up:



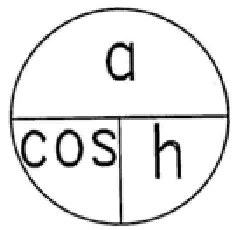
A. First draw circle as shown and enter the "o," "sin" and "h" as shown.

B. If you need the sin (angle), then cover up sin. o and h remains (opposite/hypotenuse). Because they are on different sides of the horizontal line, the answer is the product of a division.

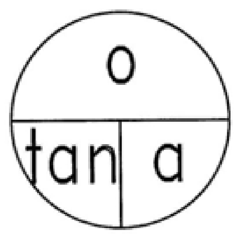
C. If you need the o, then cover up the o, and sin and h remains. Because they are on the same side of the horizontal line, the answer is the product of a multiplication (sin*h).

D. If you need the h, then cover up the h and o and sin remains (o/sin).

The process for COS is the same as SIN, draw the circle and enter a, cos and h (adjacent, cosine and hypotenuse).



The process for TAN is the same as SIN, draw the circle and enter “o” (opposite), “tan” and “a” (adjacent).

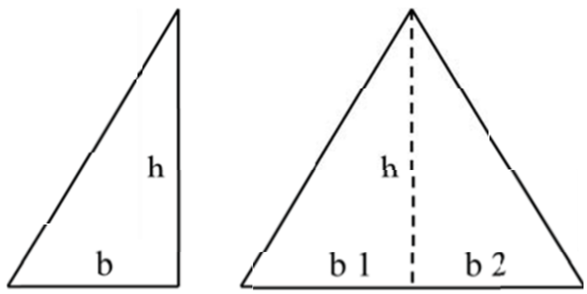


Remember, when looking for sin, cos or tan,
you are looking for an angle.

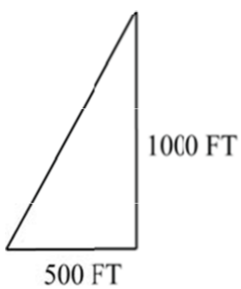
THE AREA OF A RIGHT TRIANGLE

The area of a right triangle equals one half of the product of the base and height, where the base is any side and the height is the distance measured at right angles from the base to the opposite vertex:

Given: A = Area b = base h = height 43560 square feet = 1 acre



Example 1:



Formula: $A = \frac{b * h}{2}$

$A = \frac{500*1000}{2}$

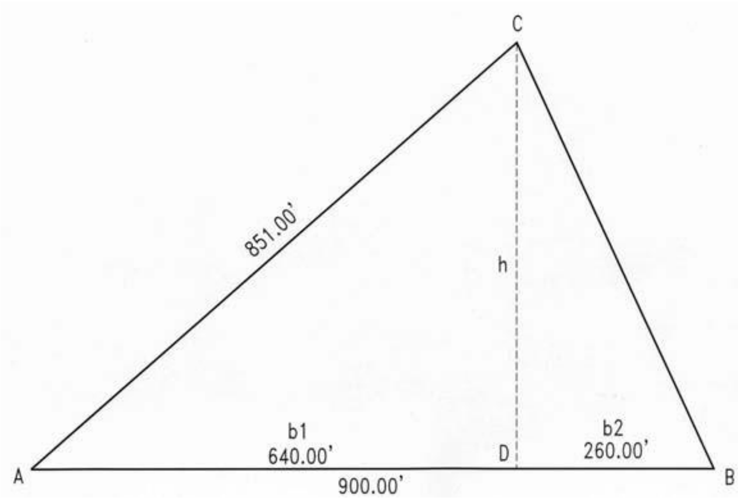
$A = \frac{500,000}{2}$

A = 250,000 square feet

A = 250,000 / 43560

A = 5.739 acres

Example 2:



h = line CD

$h = \sqrt{851^2 - 640^2}$

$h = \sqrt{724201 - 409600}$

$h = \sqrt{314601}$

h = 560.893 (answer)

ADCA Area

A = ½ (b*h)

A = ½ (640*560.893)

A = ½ (358971.5200)

A = 179485.7600 square feet

BCDB Area

A = ½ (b*h)

A = ½ (260*560.983)

A = ½ (145,832.1800)

A = 72916.0900 square feet

ABCA Area

179485.7600

+72916.0900

252401.8500 square feet / 43560 square feet per acre

Area = 5.794 acres

Check:

A = (b*h) / 2

A = (560.893*900) / 2

A = (504803.7000) / 2

A = 252401.8500 (checks) / 43560 square feet per acre

A = 5.794 acres (checks)

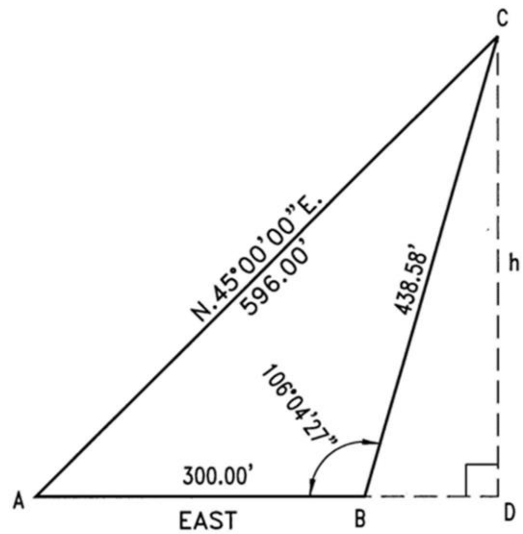
Area of a Right Triangle *(continued)*

Example 3:

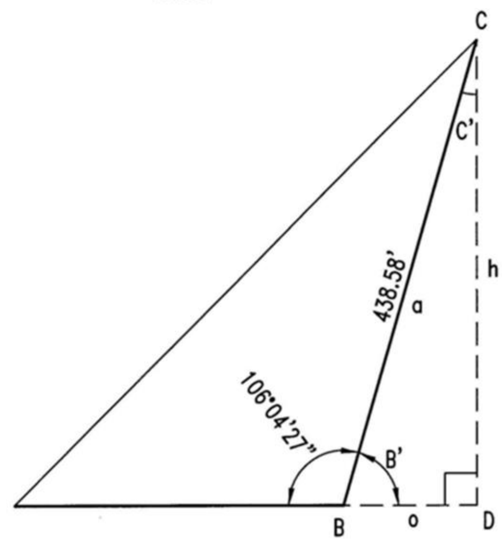
Angles:
 179º59'60" total angle size in angle (full B)
<u>-106º04'27"</u> less angle given in sketch (B)
 73º55'33" remainder angle in complete angle (B')
(see second sketch)

 90º00'00" angle given in right triangle (D)
<u>+73º55'33"</u> plus angle computed above (B')
163º55'33" total given angles

 179º59'60" total angle size in triangle
<u>-163º55'33"</u> total given angles (D & B')
 16º04'27" remainder angle



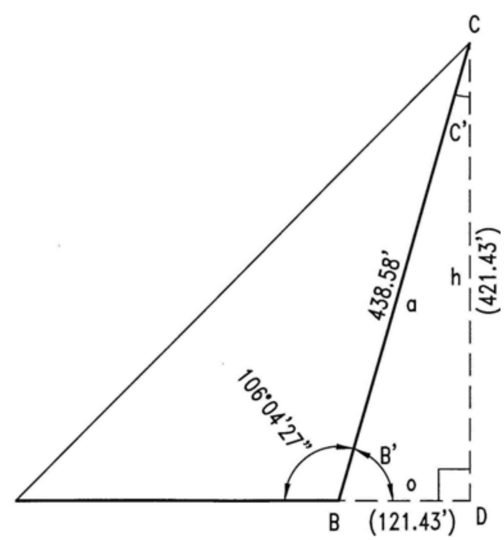
Distances:
Sin = o/h (formula) (see Page 40)
o = sin * h (a to the right, changed to match missing)
o = sin 16°04'27" * 438.58 (plug in numbers)
o = sin 16.074166 * 438.58 (change to decimal angle)
o = 0.27688143 * 438.58 (change to function of angle)
o = 121.43 (answer)



Cos = a/h (formula) (see Page 41)
h = cos * a (formula changed to match missing)
h = cos 16°04'27" * 438.58 (plug in numbers)
h = cos 16.074166 * 438.58 (changed to decimal degrees)
h = 0.96090409 * 438.58 (changed to function of angle)
h = 421.43 (answer)

Line A to D = 300.00 + 121.43 = 421.43

While we came to the correct answers on this question, we don't always have to go the LONG way around. Look at the bearings in the top sketch. They indicate that the angle at A is 45°00'00". With a plane triangle <u>always</u> equaling 180°, the missing angle at the top of the triangle (C) also has to be 45°00'00". With the hypotenuse being 596.00, the distance along "h" has to be 421.43. This also makes the distance from A to D, 421.43. or 121.43 different from that given first, the small triangle distance.



Area of a Right Triangle *(continued)*

Example 3 *Continued*:

ADCA Area
A = (b*h) / 2
A = (421.43 * 421.43) / 2
A = (177603.2449) / 2
A = 88801.6225 square feet (overall triangle)

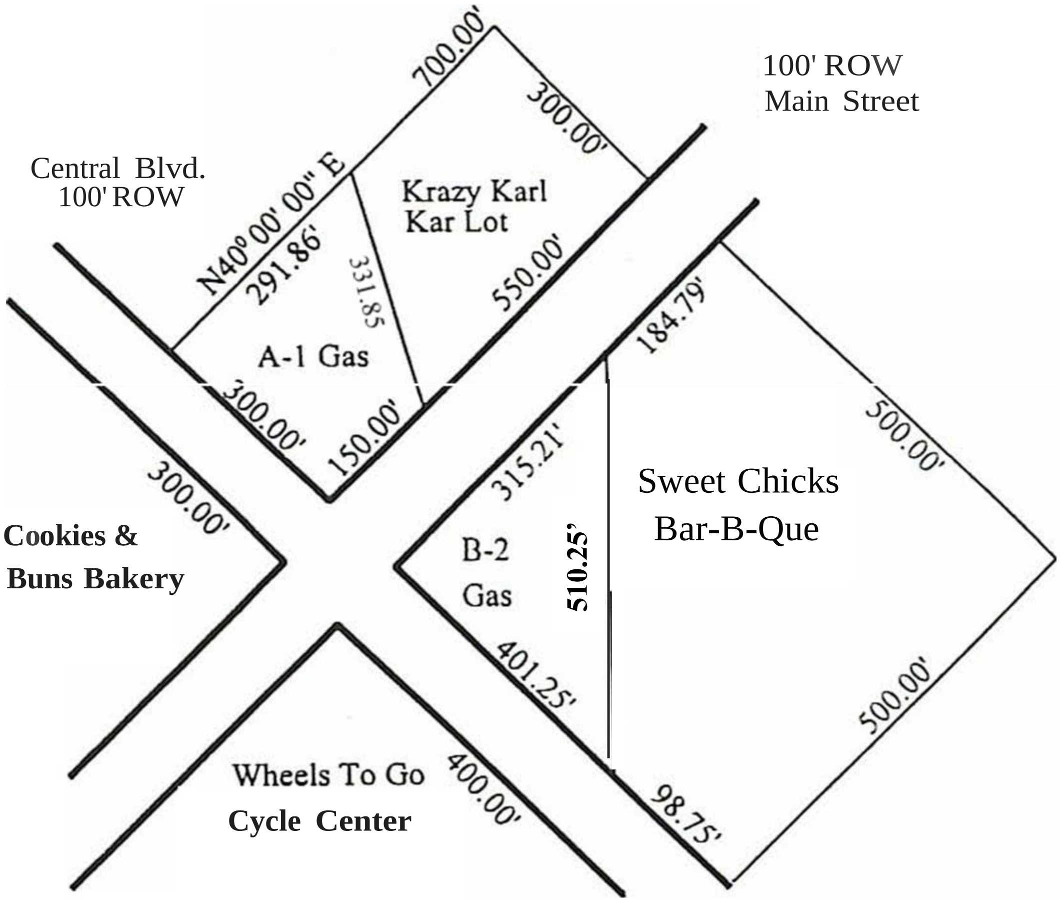
BCDB Area
A = (b*h) / 2
A = (121.43 * 421.43) / 2
A = 51174.2449 / 2
A = 25587.1225 square feet (small triangle)

ABCA area
  88801.6225 square feet, overall
- 25587.1225 square feet, small triangle
  63214.5000 ÷ 43560 = 1.451 acres, answer

Compute for the square footage first, then as the last computation, divide to get the acreage.

Area of a Right Triangle *(continued)*

Exercise:



1. Find the area of the property shown as Sweet Chicks Bar-B-Que

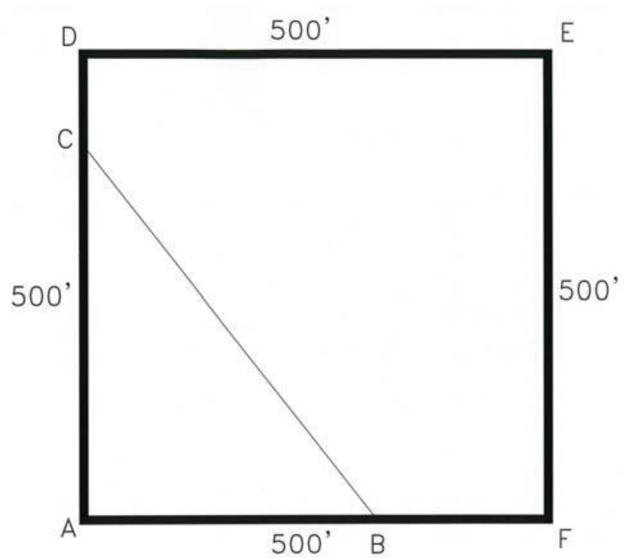
    _____________square feet ____________acres

2. Find the area of the property shown as Krazy Karl Kar Lot.

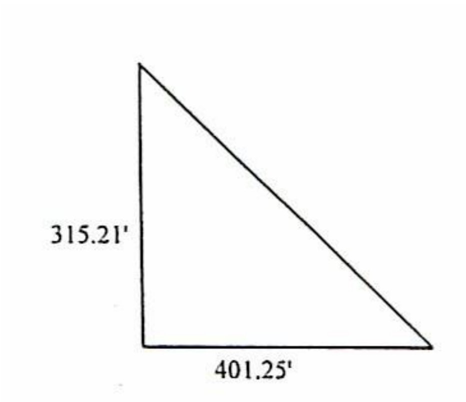
    _____________square feet ____________acres

(see next page)

Area Solutions for Sweet Chicks Bar-B-Que



Overall AFEDA = 500 * 500 = 250,000 sq. ft. = 5.739 acres



A = (b * h) / 2
A = (401.25 * 315.21) / 2
A = 126478.0125 / 2
A = 63239.00625
A = 1.452 acres (B-2 Gas parcel)

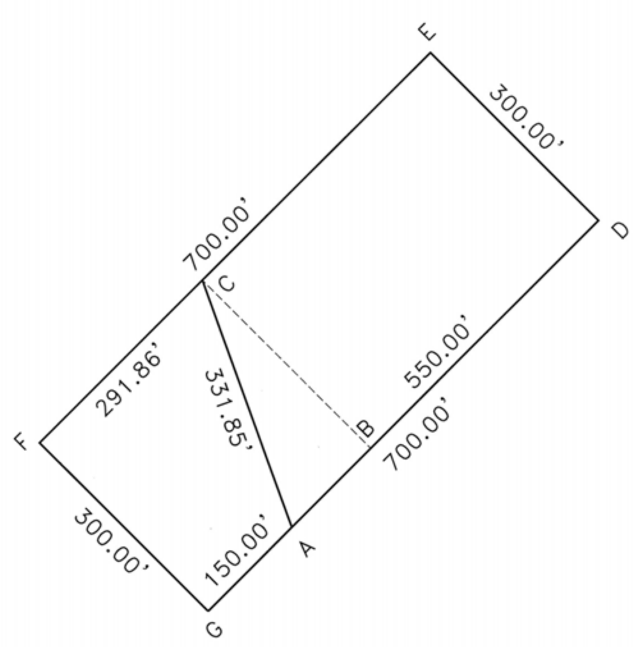
250,000.00000
-63,239.00625 (minus B-2 Gas)
186,760.99375 / 43560 =
4.287 acres (answer)
(Notice that one adjusts the square footage first, then divides to get the acreage.)

Solution for Krazy Karl Kar Lot:

(Refer to sketch on Page 50)

Get missing distances:

CE = EF-CF	AB = AD-CE
700.00'	550.00
-291.86'	-408.14
408.14’	141.86



1. Overall (GDEF): 700.00
   <u>*300.00</u>
   210,000.00 sq. ft.

2. Figure CBDE: 408.14'
   <u>*300.00'</u>
   122,442.00 sq. ft. rectangle square footage

3. Figure ABC:
   Area = (b* h) / 2
   Area = (141.86 * 300) / 2
   Area = 42,558.0000 / 2
   Area = 21,279.0000 square feet (triangle square footage)

4. Addition of square footages:
   From Item 2: 122,442.0000
   From Item 3: <u>+ 21,279.0000</u>
   Total: 143,721.0000 square feet

5. 143721.0000 / 43560 = 3.29938 acres or **3.30 acres (answer)**

Check:
Area GACF:
150.00 + 291.86 = 441.86
(441.86 * 300) / 2 = 66,279.0000 square feet
66,279.0000 + 143,721.0000 = 210,000.0000 square feet (143,721.0000 from Item 4 above)
210,000.0000 square feet = 210,000.0000 square feet (from Item 1 above)
Answer checks.

You don’t always need to solve every possible figure or triangle when computing the areas of parcels. You just have to look at the problem closely. We really didn’t have to do all the above. Look at Krazy Karl again. All we had to do for it was determine the missing distance: CE. Then ((408.14 + 550.00) / 2) * 300.00. That equals 143,721.0000 square feet. Divide by the square feet in an acre and we get 3.30 acres. The ANSWER!!!

Look closely at the way the formula is formed before just plugging in numbers. Remember what the "h" is? Solve the triangle:

Right:

$h^2 = 572.96^2 - (425.32 / 2)^2$

$h = \sqrt{328283.1616 - 45224.2756}$

$= \sqrt{283055.8860}$

$h = 532.03'$

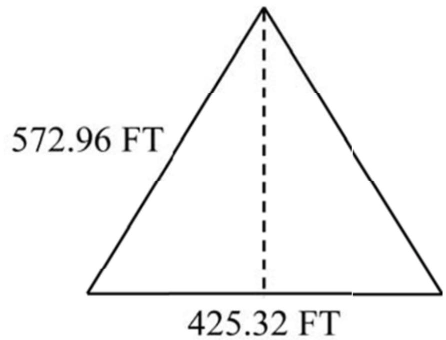
Wrong:

$h^2 = 572.96^2 - 425.32^2 / 2$

$h = \sqrt{328283.1616 - 180897.1024 / 2}$ h

$h = \sqrt{73693.0296}$

$h = 271.46'$



Area of Triangle

Area = (b / 2) * h (formulae)

A = (425.32 / 2) * 532.03) (plug in the numbers – the 532.03 comes from the above, left)

A = 113141.4998 / 43560 (square feet / square feet per acre)

A = 2.597 acres (answer)

Most of the time you only need to compute the square footage to the nearest square foot or bigger.

We are just showing you the (almost) complete square footage so you can see that it works well beyond that point.

Let us take a look at how many square feet are in various portions of an acre. Let us say 0.01 acre is 0.01 * 43560 square feet or 435.60 square feet. Let us say that 0.001 acre is 0.001 * 43,560 square feet or 43.56 square feet). So, if that many square feet are in a thousandth of an acre, one only needs to have the acreage figure within 43.56 square feet if reporting to a thousandth of an acre and only 435.60 square feet if reporting to a hundredth of an acre. Remember this when figuring acreage.

Using the Sketch Below, solve for the following questions:

1. Find the area of the property shown as Tom's Pet Cemetery
   ___________Square Feet ___________Acres

2. Find the area of the property shown as Jerry's Pet Emporium
   ___________Square Feet ___________Acres

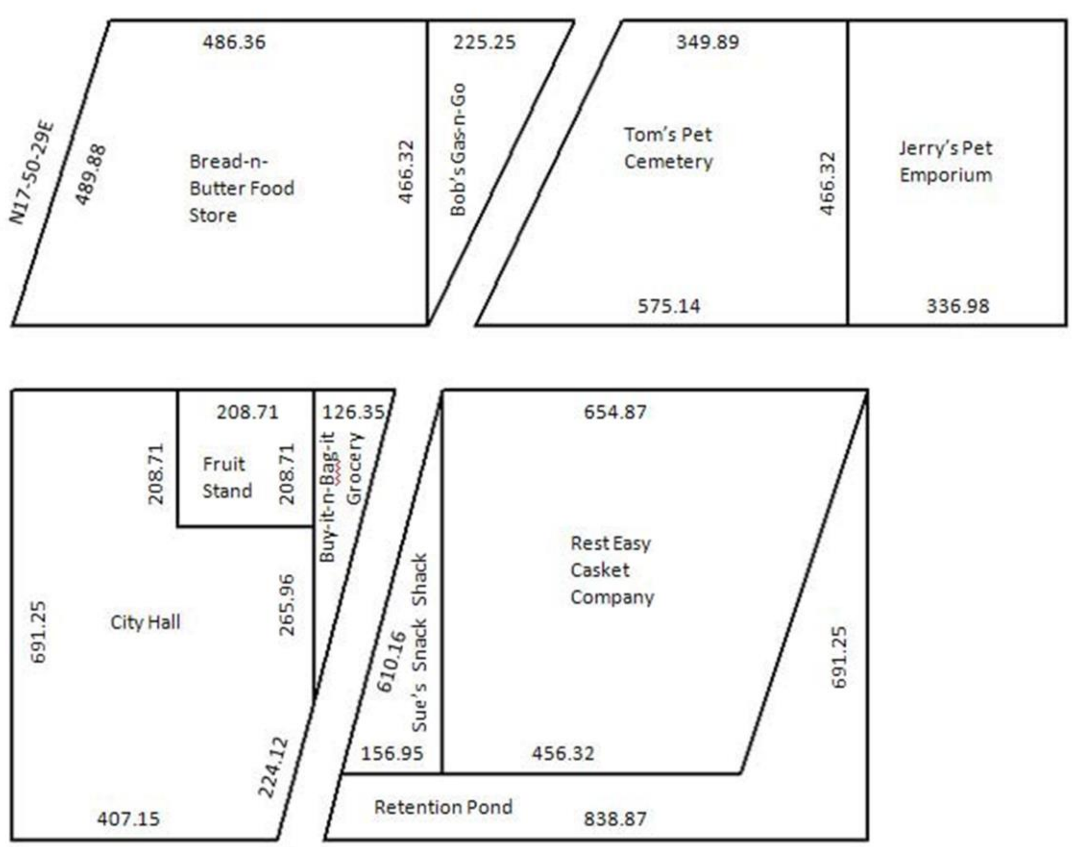
3. Find the area of the property shown as City Hall
   ___________Square Feet ___________Acres

4. Find the area of the property shown as Bread-n-Butter Food Store
   ___________Square Feet ___________Acres

5. Find the area of the property shown as the Fruit Stand
   ___________Square Feet ___________Acres

6. Find the area of the property shown as Buy-it-n-Bag-it Grocery store
   ___________Square Feet ___________Acres

7. Find the bearings along the Right-of-Way for the parcels below:
   ___________Bob's Gas-n-Go ___________Sue's snack Shack



Other Methods

To go into other methods of triangle areas, you have to get into the area in ANY triangle, not just right triangles. The area of ANY triangle is equal to ½ ab * Sin I where I is any included angle and a and b are the included sides. As commonly written: Area = ½ ab * Sin I. Another method when all you have is the sides on ANY triangle is s = ½ (a + b + c). This is the factor for the formulae: Area = $\sqrt{s(s-a)(s-b)(s-c)}$.

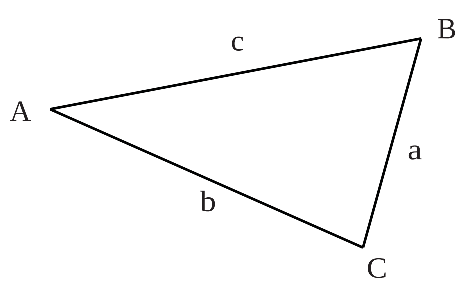
We will not discuss the methods above further in this course, but you now know that they exist and may be used when one of the other methods are too involved to use easily.

INTERMEDIATE PRINCIPLES OF TRIGONOMETRY

There are basically two triangles that are the basis of the study of Trigonometry. These are known as the right triangle and oblique triangle (the acute triangle is another form of the oblique in that the angles are not greater than 90º and the obtuse triangle where one of the three angles is greater than 90º). We have already discussed the right triangle which is the key to the whole system of computations of three sided polygons. We will now discuss the oblique triangle. The oblique triangle uses the same principles as the right triangle, but adds some new rules as we shall see below.

Oblique triangles

Oblique triangles are triangles that do not have a ninety degree angle. These triangles can be solved by subdividing them into a set of right triangles. Fortunately, mathematicians have done this for us and have given us a set of formulas for solving this type of problem. Remember, however, that any oblique triangle can be solved by making several right triangles out of it. If we are given a typical oblique triangle as shown below the formulas are as follows.



THE LAW OF SINES:

$$\frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

THE LAW OF COSINES:

$c^2 = a^2 + b^2 - (2ab \cos C)$

With the basic right triangle problems and the two laws stated above, <u>most</u> of the cadastral mapping problems can be solved.

$$\frac{a-b}{a+b} = \frac{\tan(\frac{\alpha-\beta}{2})}{\tan(\frac{\alpha+\beta}{2})}$$ There is also a Law of Tangents which can be used.

Now that we have some of the basic formulas established, let's attempt some problems. The oblique triangle shown previously will be the figure to which these problems are related.

Problem 1.

If **angle C** = 42º 32' 48", and **angle B** = 31º 54' 12", and **side a** = 125.34', what are the remaining values of the triangle? Find the area.

b =

c =

A =

Area =

Problem 2.

If **angle A** = 108º 23’ 32”, and **side a** = 234.56,’ and **side b** = 197.35’, what are the remaining values of the triangle? Find the area.

c   =

B =

C =

Area =

Problem 3.

If **angle B** = 35º 37’ 23”, and **side a** = 212.67’, and **side c** = 274.89’, what are the remaining values of the triangle? Find the area.

A =

C =

b =

Area =

Problem 4.

If **side a** = 289.12', **side b** = 175.24', and **side c** = 212.34', what are the other values of the triangle? Find the area.

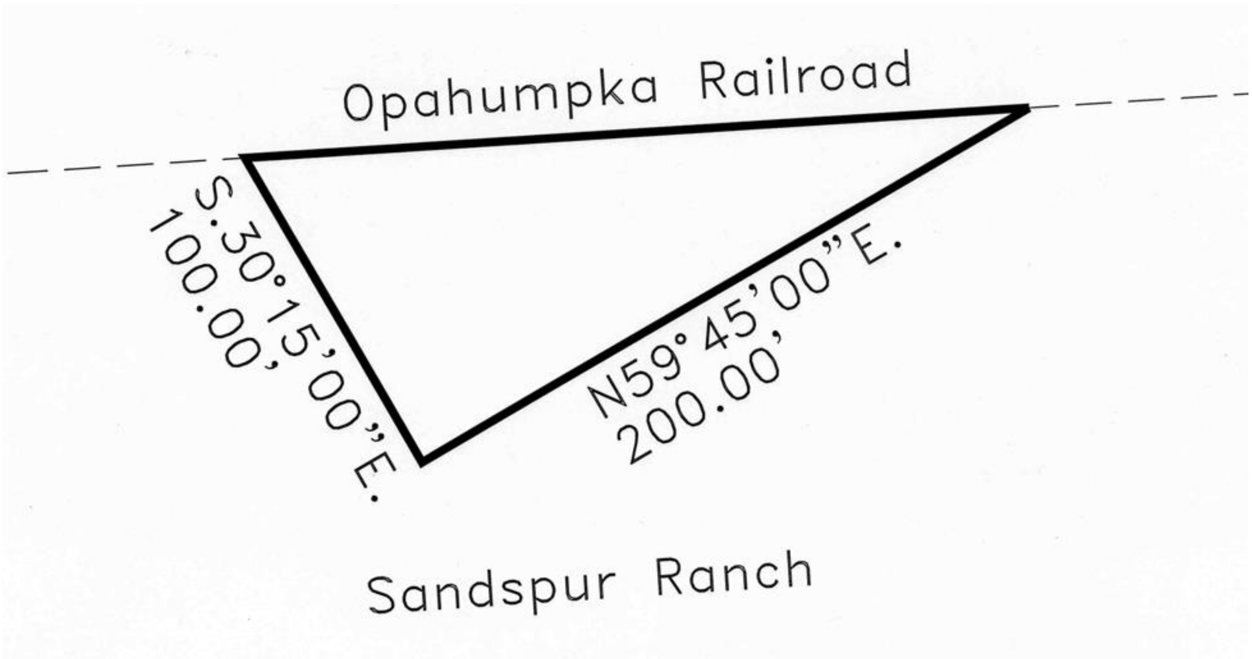
A =

B =

C =

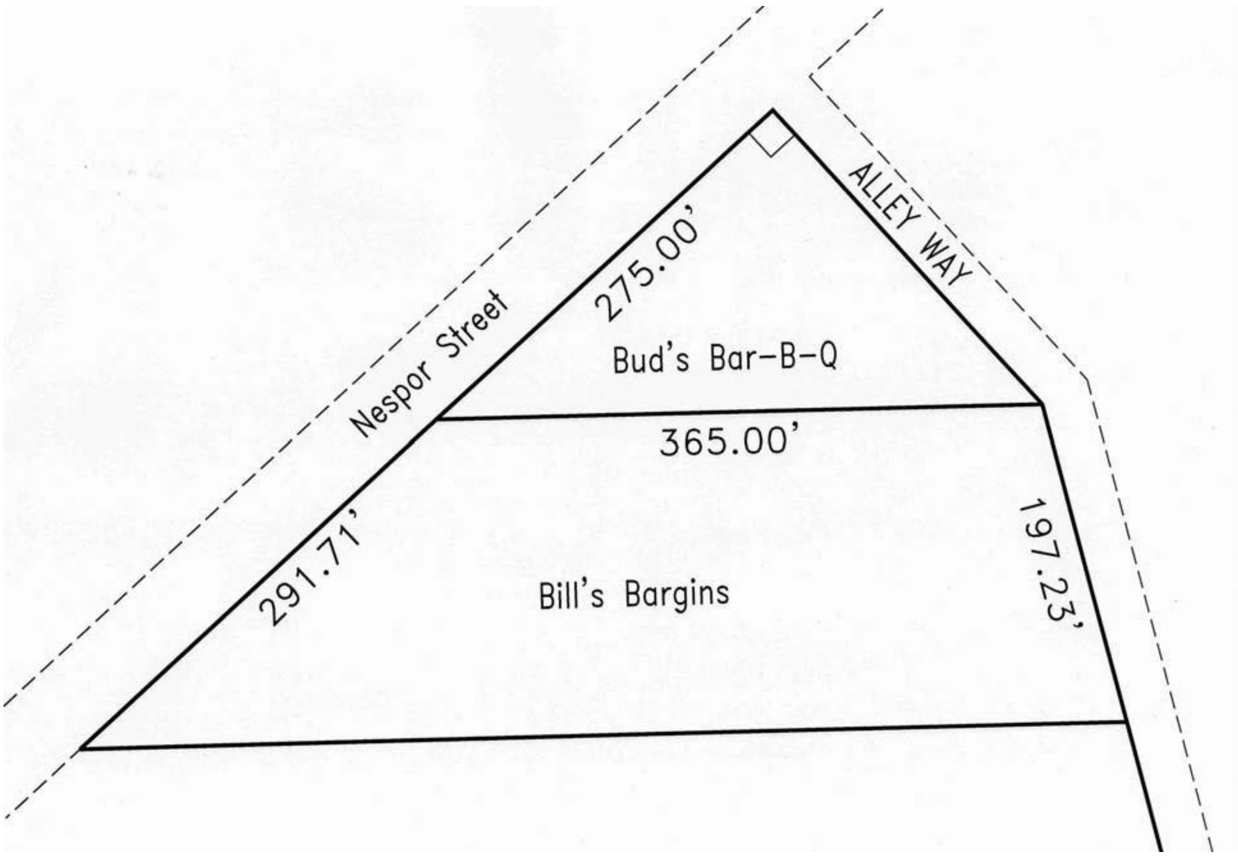
Area =

TRIGINOMETRY PRACTICAL APPLICAITONS



The Opahumpka Railroad has acquired a parcel of land for a cattle loading station for the Sandspur Ranch. Your assignment is to calculate the distance along the railroad frontage and calculate the acreage.

The City of Dumpster has condemned and acquired by fee simple title, the parcel shown below called Bud's Bar-B-Q. Your assignment is to compute the rear lot line and the acreage for the Bud's Bar-B-Q parcel.



Lot line =

Acreage = ______________________________

"Begin at the Northwest Corner of Section 2, Township 3 North, Range 29 West, Gull County, Florida; thence South along the West line of said Section 2, a distance of 1560.00 feet to the Point of Beginning: thence S25º45'00"E, a distance of 500.00 feet; thence N50º20'00"E, a distance of 800.00 feet to the South Line of the Opahumpka Railroad; thence Southwesterly along the South line of the Opahumpka Railroad to the Point of Beginning."

Your assignment is to compute the distance along the railroad frontage and the acreage.

Work Area:

Day 2 Practice

1. 88^2 = ____________________________

2. 156^2 = ____________________________

3. 426^2 = ____________________________

4. 325^2=____________________________

Round below square roots to nearest hundredth

5. $\sqrt{1805}$ = ___________________________

6. .$\sqrt{2406}$ = ___________________________

7. $\sqrt{529}$ = ____________________________

8. $\sqrt{5643}$ = ____________________________

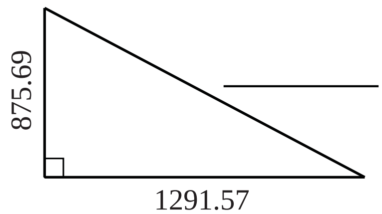
Convert the numbers given to the inverse of the associated Trig Function, then convert from the DD (decimal Degrees) angle to DMS (Degrees Minutes Seconds) angle.

	Trig Value:	Decimal equivalent	Angle equivalent:
sine	9) .999946981		
	10) .094668186		
	11) .724489404		
cosine	12) .691471029		
	13) .945803901		
	14) .167743834		
tangent	15) 5.299955524		
	16) .583319208		
	17) .370738184		

Day 2 Practice

Exercise: Find the missing side and the area in square feet.

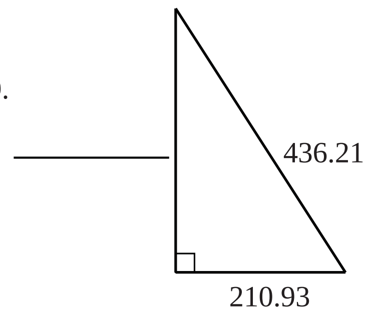
18.



Side ______________

Area ______________

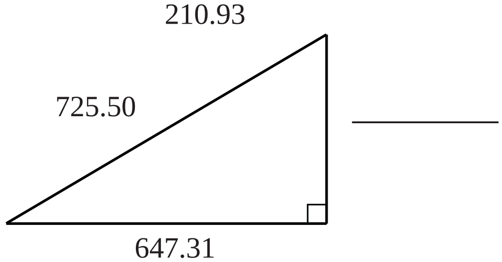
19.



Side ______________

Area ______________

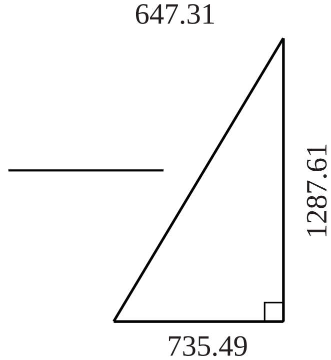
20.



Side ______________

Area ______________

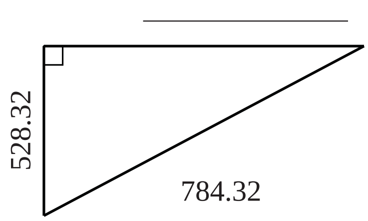
21.



Side ______________

Area ______________

22.



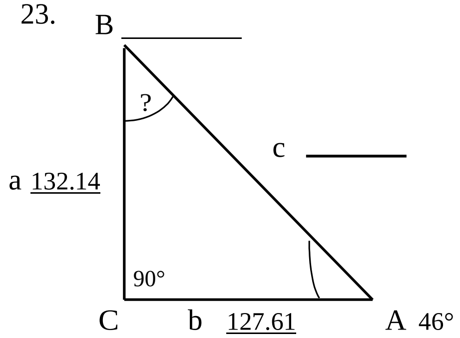
Side ______________

Area ______________

Day 2 Practice

Exercise: Find the missing side, angles, and area of each right triangle.

23.

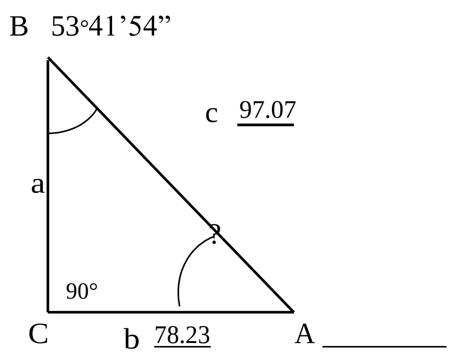


Angle B ______________

Side c ______________

Area ______________

24.

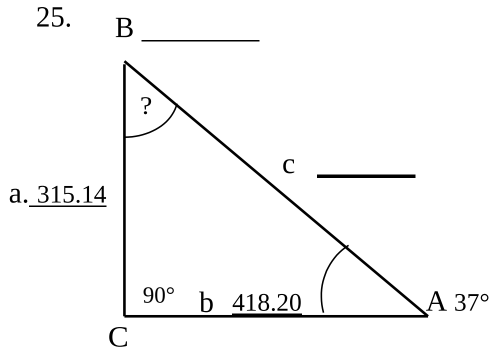


Angle A______________

Side a ______________

Area ______________

25.



Angle B ______________

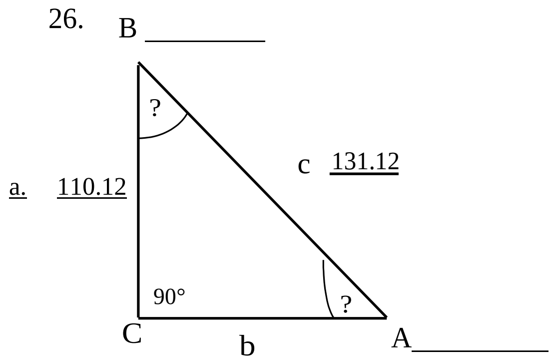
Side c ______________

Area ______________

Day 2 Practice

Exercise: Find the missing side, angles, and area of each right triangle.

26.



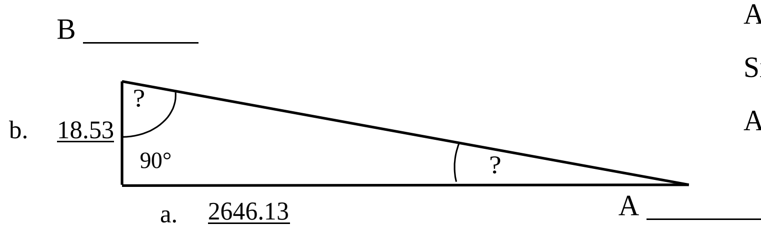
Angle A ______________

Angle B ______________

Sideb________________

Area_________________

27.



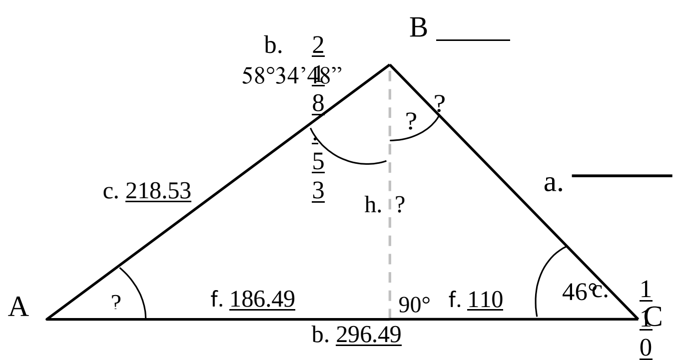
Angle A ______________

Angle B_______________

Side c_______________

Area _________________

28.



Angle A__________________

Angle B (Total)___________

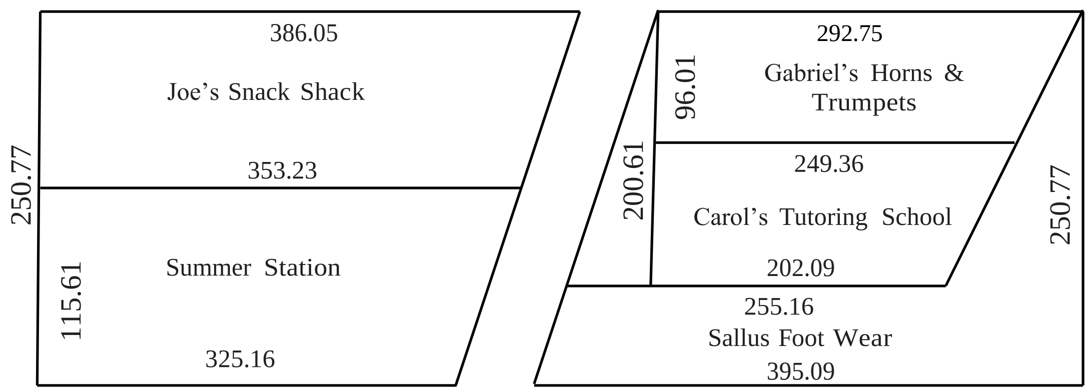
Side h _____________________

Side a ____________________

Area _____________________

Day 2 Practice

Exercise: Determine the area of these properties.



29. Find the area of the property shown as Carol's Tutoring School.

___________square feet ____________acres

30. Find the area of the property shown as Joe's Snack Shack.

___________square feet ____________acres

31. Find the area of the property shown as Gabriel's Horns & Trumpets.

___________square feet ____________acres

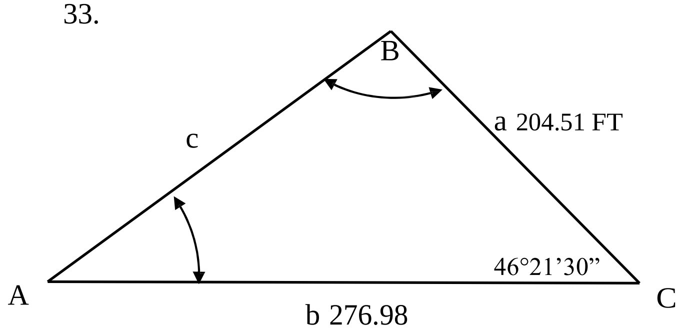
32. Find the area of the property shown as Summer Station.

___________square feet ____________acres

Day 2 Practice

Exercise: Find the missing side, angles, and area of each oblique triangle.

33.



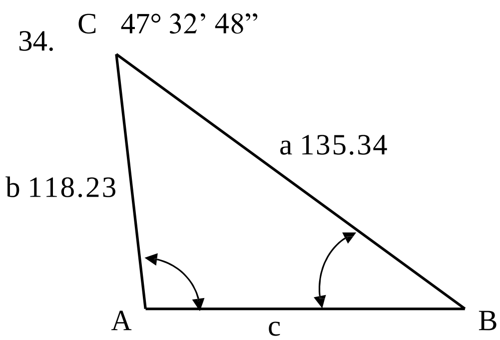
Angle A________________

Angle B________________

Side c __________________

Area __________________

34.



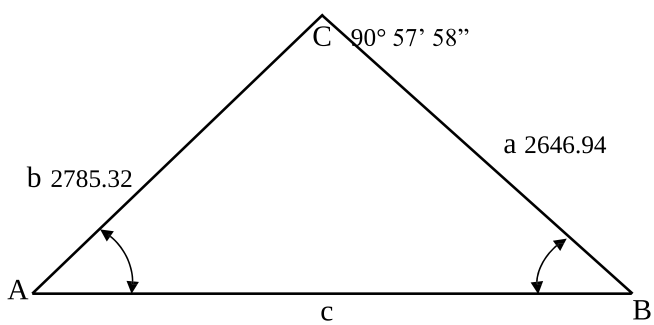
Angle A________________

Angle B________________

Side c___________________

Area ___________________

35.



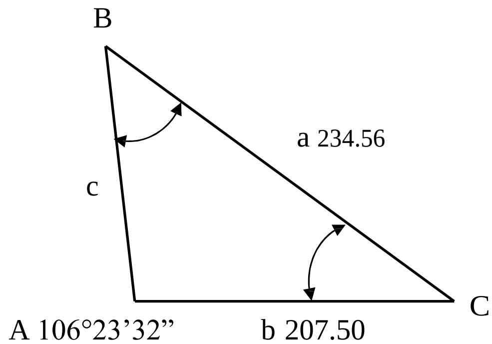
Angle A ________________

Angle B________________

Side c __________________

Area ___________________

36.



Angle B________________

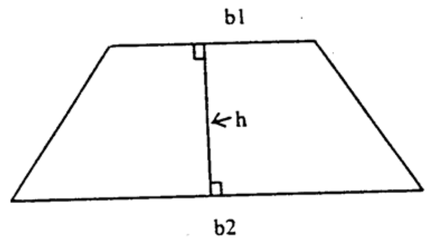
Angle C ________________

Side c __________________

Area ___________________

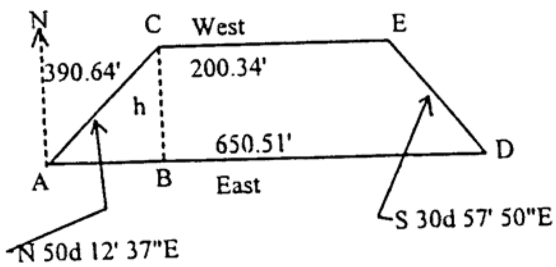
The Trapezoid

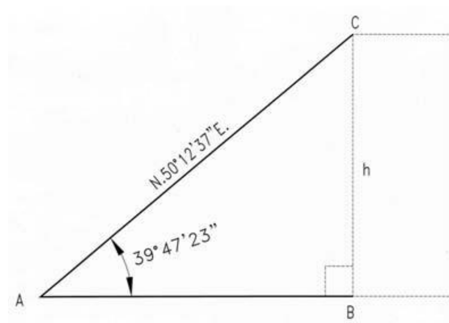
The area of a trapezoid is one half the sum of the bases times the height. This is based on the bases being parallel and the height being perpendicular to the bases. Each trapezoid has two bases really. Notice one on the top and one on the bottom of the sketch (b_1 and b_2).



$$A = ½ (b_1 + b_2) * h$$

Trapezoid Example:





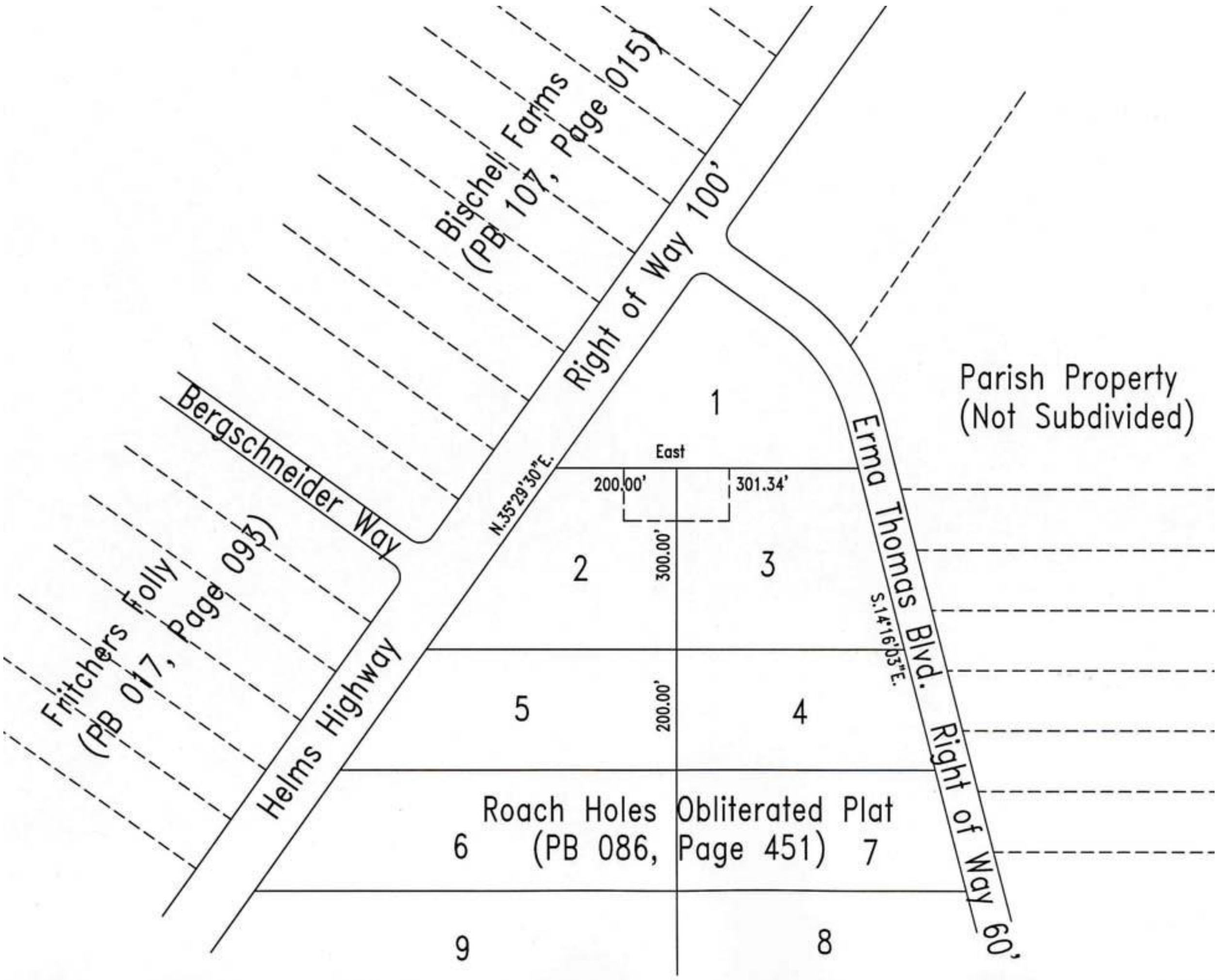
Angle CAB:	Side h:
89º 59' 60"	sin = o/h (formulae)
-50°12' 37"	o = sin * h (change formulae to match existing numbers, see Page 40)
39° 47'23"	o = sin 39°47'23" * 390.64 (plug in the numbers)
	o = sin 39.78972222 * 390.64 (change angle to decimal angle)
	o = 0.63997187 * 390.64 (get the sin of the decimal angle)
	o = 249.9986 or 250.00 (multiply and get answer)

Area ADECA:

$A = ½ (b_1 + b_2) * h$
A = ½ (200.34 + 650.51) * 250
A = ½ * 850.85 * 250
A = 106356.25 square feet / 43560 square feet per acre
A = 2.44 acres, answer

The Trapezoid Problem in Zip County

Find the combined area of Lots 2 and 3, Block 7, Roach Holes Obliterated Plat, Plat Book 086 on page 451 of the Public Records of Zip County, Florida.



Working Area:

See the sketch below:

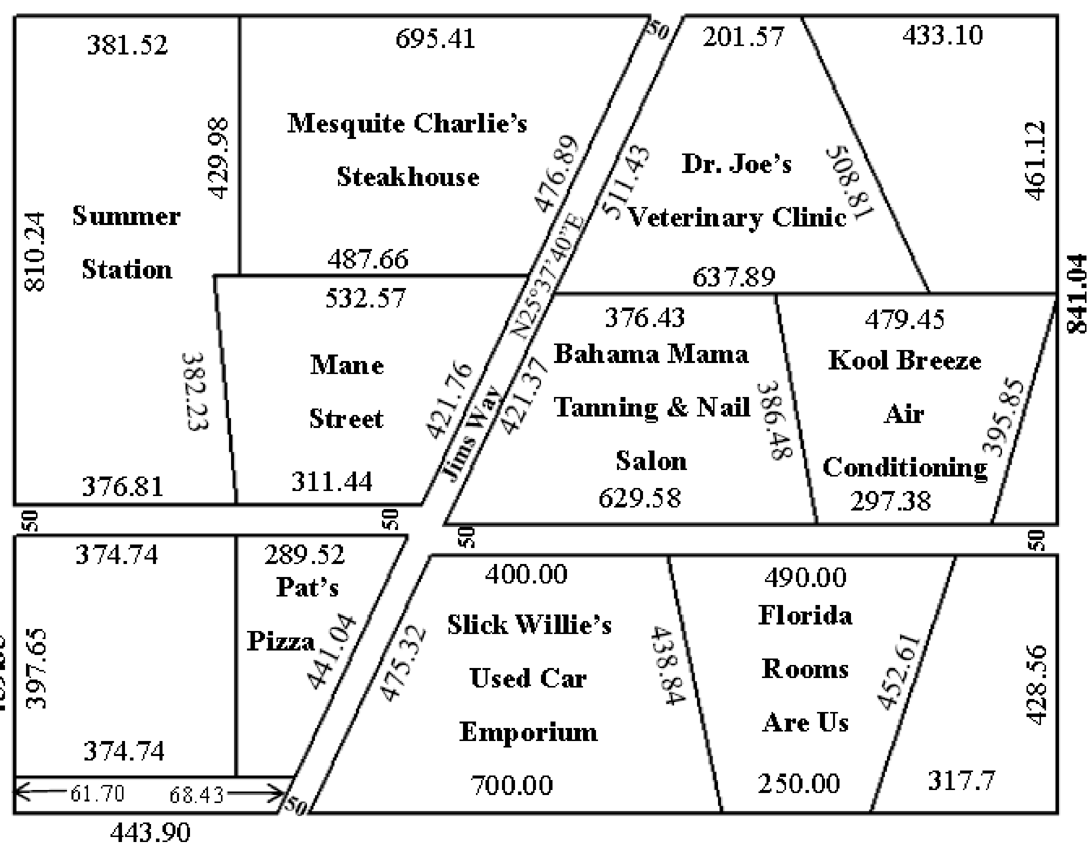
1. Find the area of Slick Willie's Used Car Emporium:
_____________square feet _____________acres

2. Find the area of Bahama Mama Tanning & Nail Salon:
_____________square feet _____________acres

3. If Jim's Way is 50 FT wide, what is the distance from Mesquite Charlie's Steakhouse NE corner to Dr. Joe's Veterinary Clinic NW corner?
____________Distance

4. Find the area of Dr. Joe's Veterinary Clinic:
_____________square feet _____________acres

5. Find the area of Mesquite Charlie's Steakhouse:
_____________square feet _____________acres



COORDINATE

History:

In 1637 a gentleman, Rene Des Cartes, published a book which he called GEOMETRY. In this book, he described a system of using a set of "axes" as a method for computing linear and curve linear formulae. There are two axes in this plane, or two dimensional system: the "x" axis and the "y" axis. With this, a revelation of a new era in mathematics was opened up. The system came to be known as the "Cartesian Coordinate System," but is often just referred to as a rectangular coordinate system. The system allows the location of a set of points from an origin at the intersection of the axes. Higher level mathematics would be difficult, if not impossible, if there were no Cartesian Coordinate System. Suffice it to say that this was indeed a giant step forward for science and math.

To jump many hundreds of years ahead, let us look at several coordinate systems that are in used in the world. You may have been first introduced to coordinates in grade or high school geography class.

Established Systems:

This would have been an introduction to latitude and longitude. Latitude measures a position from the equator and longitude measures a position from the zero meridian at Greenwich, England. To the global navigator or a Floridian plotting hurricanes, this coordinate system is very important. It is probably not called a coordinate system, but in fact, it is. The difference between this system and a regular coordinate system is that this system matches the curve of the earth whereas a true coordinate system is a plane type.

The more arcane and less well known to the layman is the Universal Transverse Mercator (UTM). If you have ever studied quadrangle maps (see the two examples), you have probably noticed the tic marks and weird numbers at the edges of the map that represent these coordinates. These coordinates cover the earth from the equator to eighty degrees north and eighty degrees south latitude. They are set up in zones or the best possible fit from a spheroid to a plane surface that are 15 degrees wide. The map projection that is utilized in this system is the Mercator, which is a cylindrical projection.

Another system that you probably have some knowledge of is the Florida State Plane Coordinate System. This system is also based upon map projections that have been developed to indicate positions on a "plane" surface that exists in reality on a three dimensional, multi-curved earth. The Florida State Plane Coordinate System is divided into three parts because of the shape of the state.

The peninsula is divided into an East and a West Zone. These zones are based upon the cylindrical Mercator Projection. The panhandle region is in what is called the North Zone and is based upon the Lambert Conic Projection. The coordinates have been computed so that they represent a two dimensional system with a grid north as a bearing base normally set in the center of the grid. To translate these directions and distances to the surface of the earth requires some math and an understanding of the system which we won't get into in this course.

State Plane Coordinate System and a Typical Surveyor' Coordinate System:
Surveyors also use a coordinate system when calculating their plats and surveys. The only difference between a surveyor's coordinate system and a state plane coordinate system is:

a. chances are a surveyor's coordinate system is not based on anything like state plane coordinates are although they could be and
b. surveyors typically use N(orth) and E(ast) or y and x in their coordinate system rather than in the x and y order. They are the same (x equals an eastern coordinate and y equals a northern coordinate).

While state plane coordinates have error in them (it is a known error), it is limited to a ratio of 1:10000 so surveyors and others can use them with confidence.

Rules for Use:

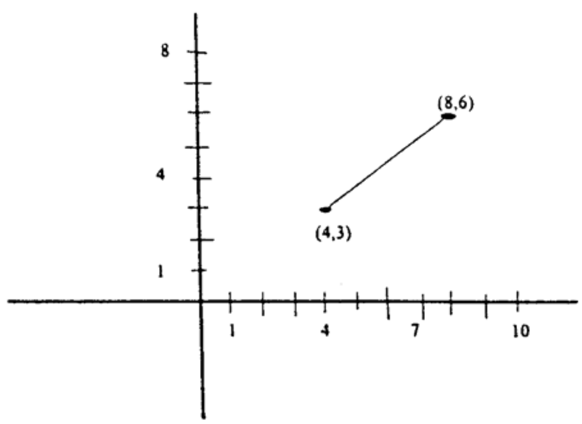
As alluded to in the previous sections, the <u>plane</u> coordinate system is based on two dimensions, one from an "x" axis and one from a "y" axis. Any point can be so identified. In the mathematical uses, there are positive and negative coordinates since the axis extends through the origin in the negative direction. For earth related coordinates, the origins have been carefully selected so that all values for the coordinates are positive. Both bearings and azimuths can be used in coordinate systems, but for real property, the bearing is far and away the choice for the direction component. If we had to go to the origin each time, the coordinate system would be a cumbersome system. The key is <u>the difference of the distances from the origin.</u>

To find the distance between the two points, we find the differences between the x's and y's, or 8 – 4 = 4 and 6 – 3 = 3 (see sketch below). This tells us that the points are separated in the easterly direction by four units and in the northerly direction by three units. From basic geometry, we know that the direct distance between these two points is five units (the 3, 4, 5 triangle rule).

If we were to state this principle in mathematical terms we would say:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = d^2$$

This, of course, is the Pythagorean Theorem in another form. Possibly you have seen this formula appear in different forms in <u>all</u> of our work. They are the foundation for many more complex and sophisticated algorithms.



ANGULAR RELATIONSHIP

As a problem, try to figure how we could derive a bearing for the direct line between the shown points. A clue is to use the tangent function.

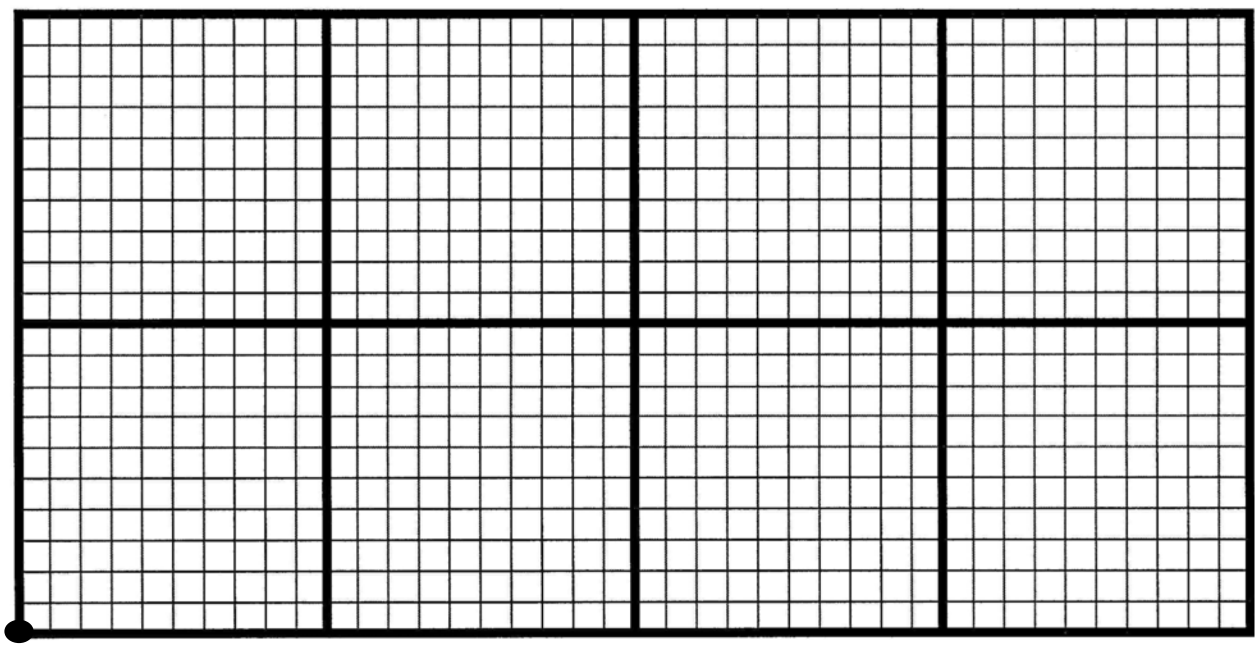
The rule for the bearing is:

$(x_1 - x_2) \;/\; (y_1 - y_2) = \tan \theta$

where theta is the bearing angle.

Point Location Using Coordinates

To locate a point using this system, it is necessary to know both the X and Y values. The X value establishes the position East of zero, and the Y value establishes the position North of zero.



X = 0, Y = 0

Example description by Coordinates:

PT 1:	X = 1200	Y = 500
PT 2:	X = 300	Y = 1100
PT 3:	X = 1000	Y = 1500
PT 4:	X = 1500	Y = 1300
PT 5:	X = 1200	Y = 500

Plot the points to the left on the above grid.
Number the grid with suitable figures.

Example Questions:

1. Using the distance formula to find the distance d, between the following points (3,7) and (2,5).

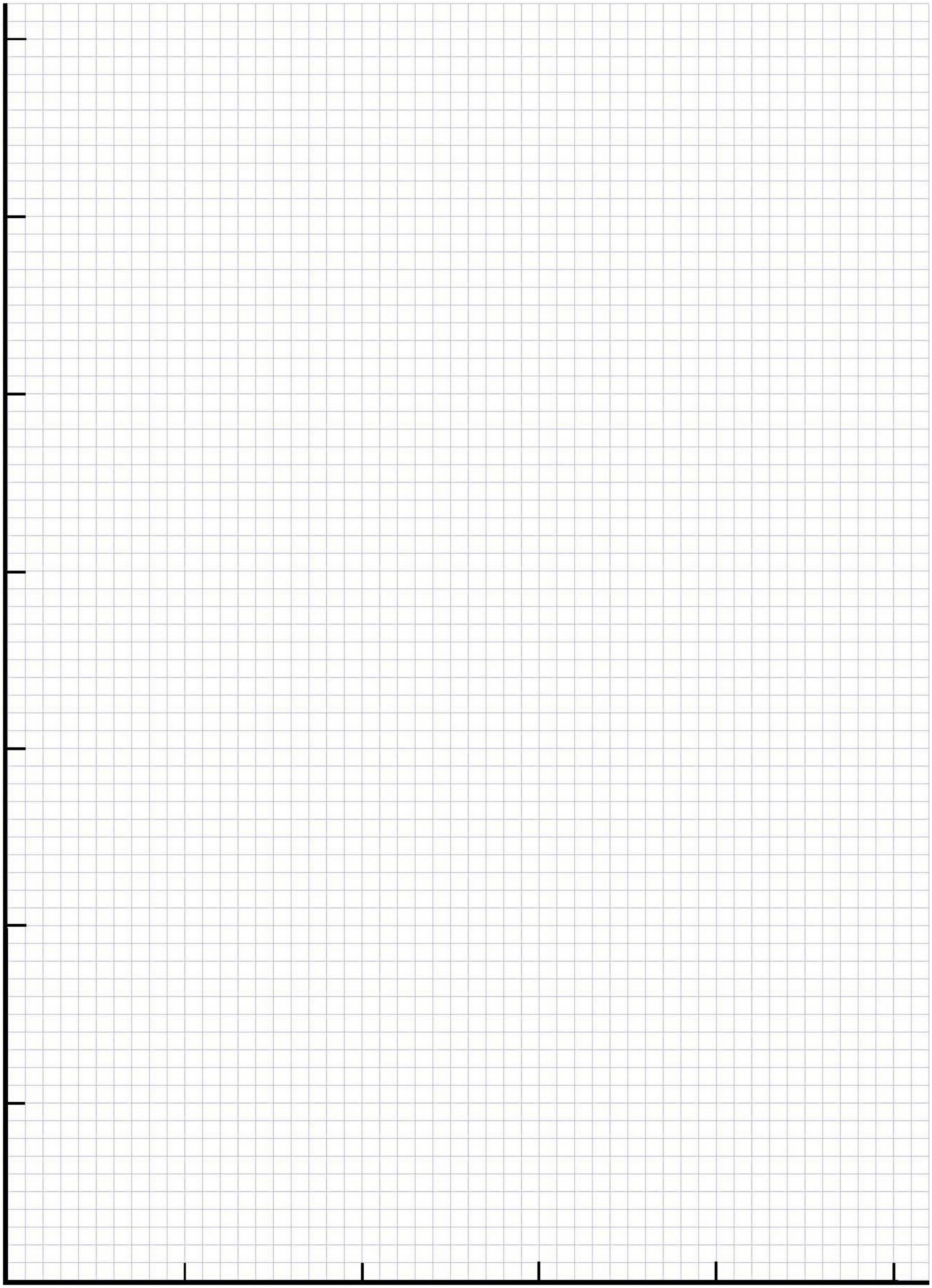
2. Using the distance formula to find the distance d, between the following points: (1,4) and (3,6).

3. Using the distance formula to find the distance d, between the following points: (-3,8) and (-2,-8).

4. Find the bearing angles in the preceding problems.

5. Using the following grid, plot the following three property descriptions (a, b, c) from their coordinates, and connect the points to enclose each property. Get the distances around the plots. Number the grid with suitable figures. Also use different colors.

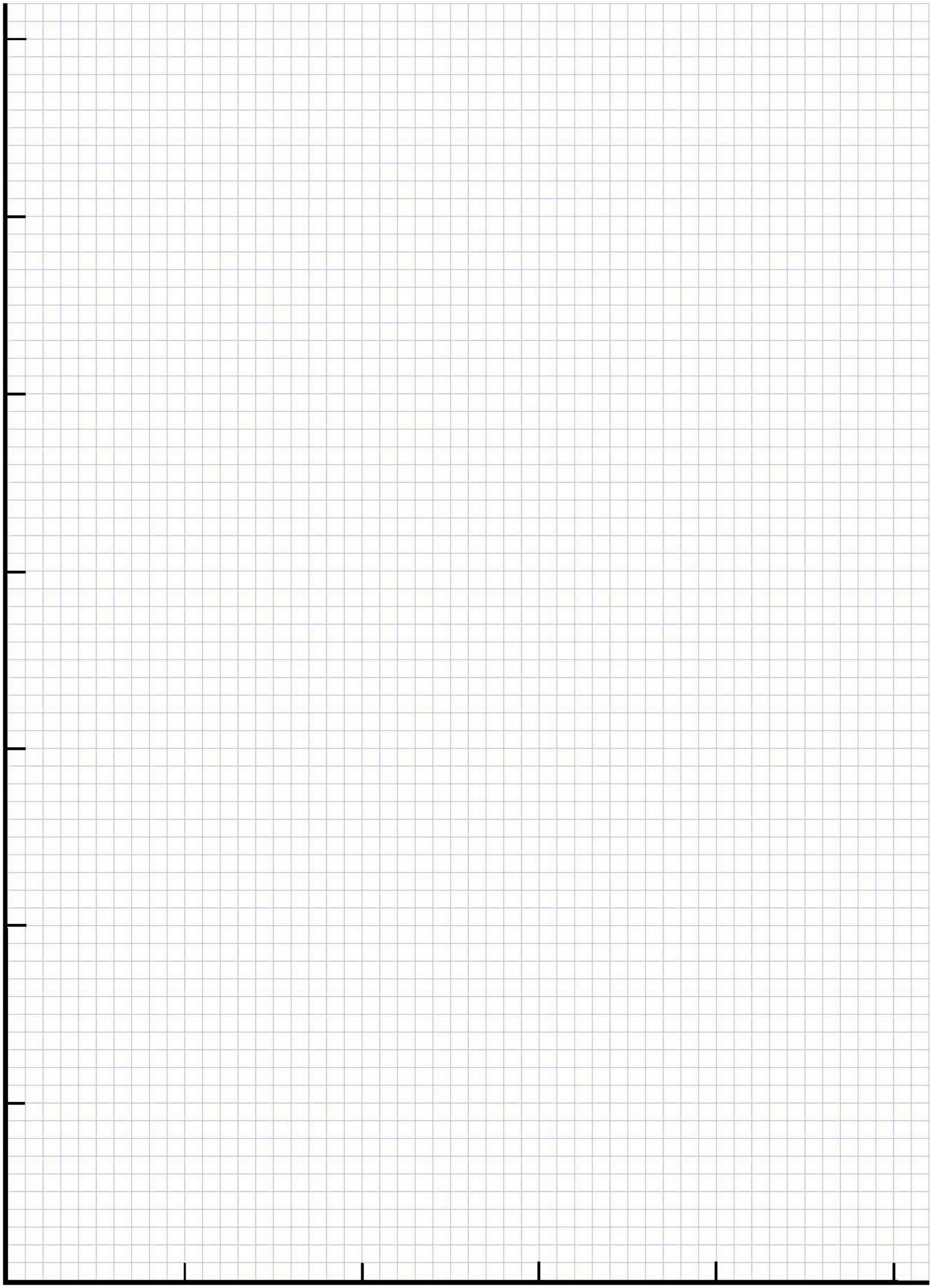
a)	b)	c)
N 200	N 440	N 30
E 80	E 380	E 240
N 480	N 520	N 130
E 220	E 440	E 300
N 400	N 440	N 200
E 320	E 500	E 480
N 280		N 150
E 360		E 490
N 220		
E 240		



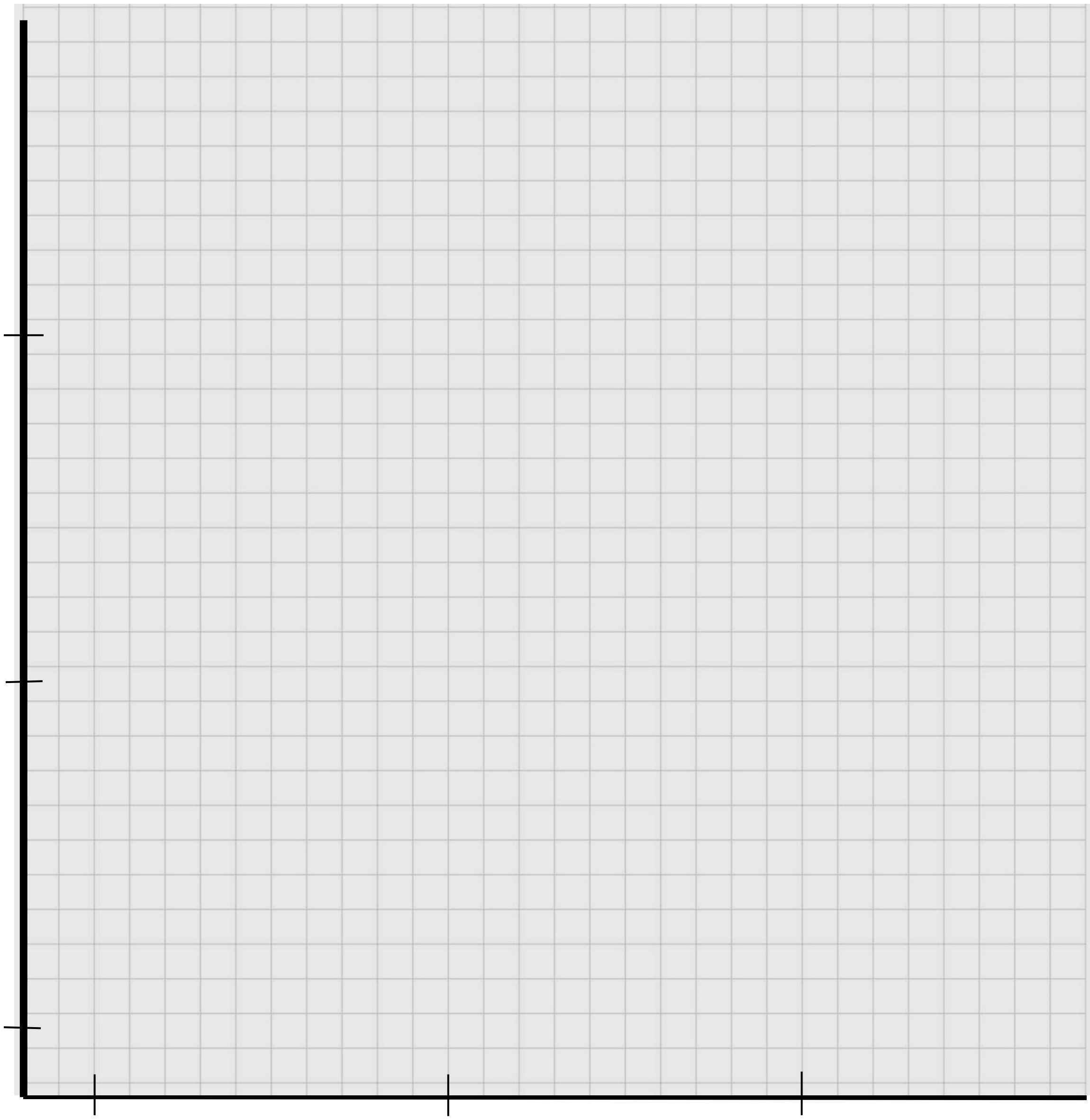
6. The Gem Oyster Company has leased a section of Bahamas Bay described as follows:

BEGINING at a post which is the Southeast Corner of Section 25, Township 3 South, Range 28 West, Bambini County, Florida, with the following coordinates: X coordinate of 500,000 and the Y coordinate of 500,000; thence North along the East line of said Section 25 to an X coordinate value of 500,000 and a Y coordinate of 501,000; thence West to an X coordinate of 497,360 and a Y coordinate of 501,000 along a line that is parallel to the South Line of said Section 25 to the half section line off said Section 25; thence South to an X coordinate of 497,360 and a Y coordinate of 500,000 to the South Quarter Corner of said Section 25; thence East along the South Line of said Section 25 to the POINT OF BEGINNING.

Your assignment is to number the above grid with suitable numbers, plot the above description on the grid and determine new coordinate values for the corners of the West 600 feet of the East 1000 feet of the South 500 feet for a cutout of the Gem Oyster Company lease.



Practice - Coordinates



(**X** = 59,800.00, **Y** = 800.00)

Calculate the coordinates for the following Bearing & Distance starting from the coordinates of **X** = 60,000.00 and **Y** = 1,000.00:

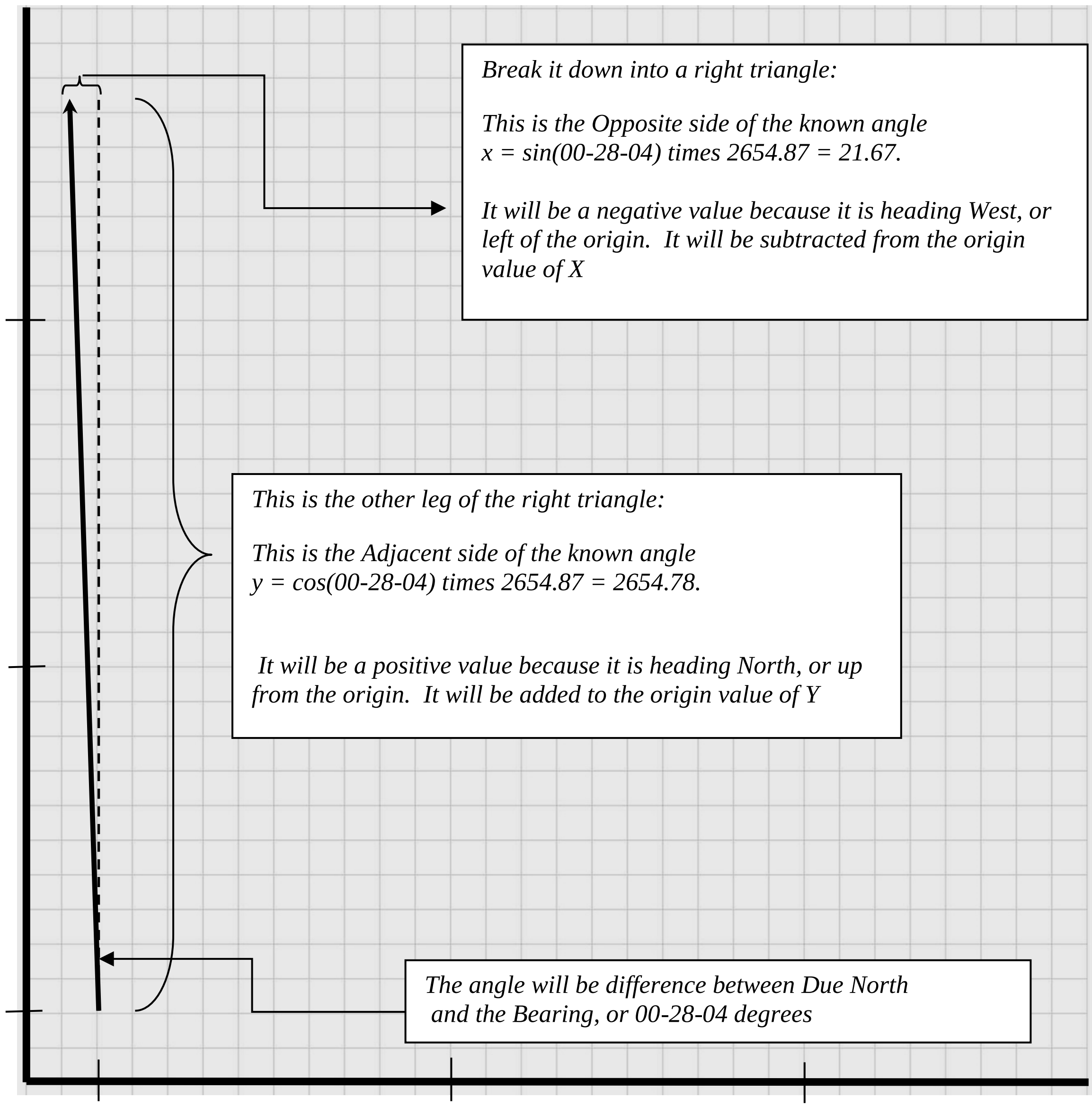
N00-28-04W 2654.87 feet

1.) What is the value of the x difference; is it pos. or neg. and why?

2.) What is the value of the y difference; is it pos. or neg. and why?

3.) What is the coordinate at the end of the call?

Practice – Coordinates Solution



(X = 59,800.00, Y = 800.00)

Calculate the coordinates for the following Bearing & Distance starting from the coordinates of **X** = 60,000.00 and **Y** = 1,000.00:

N00-28-04W 2654.87 feet

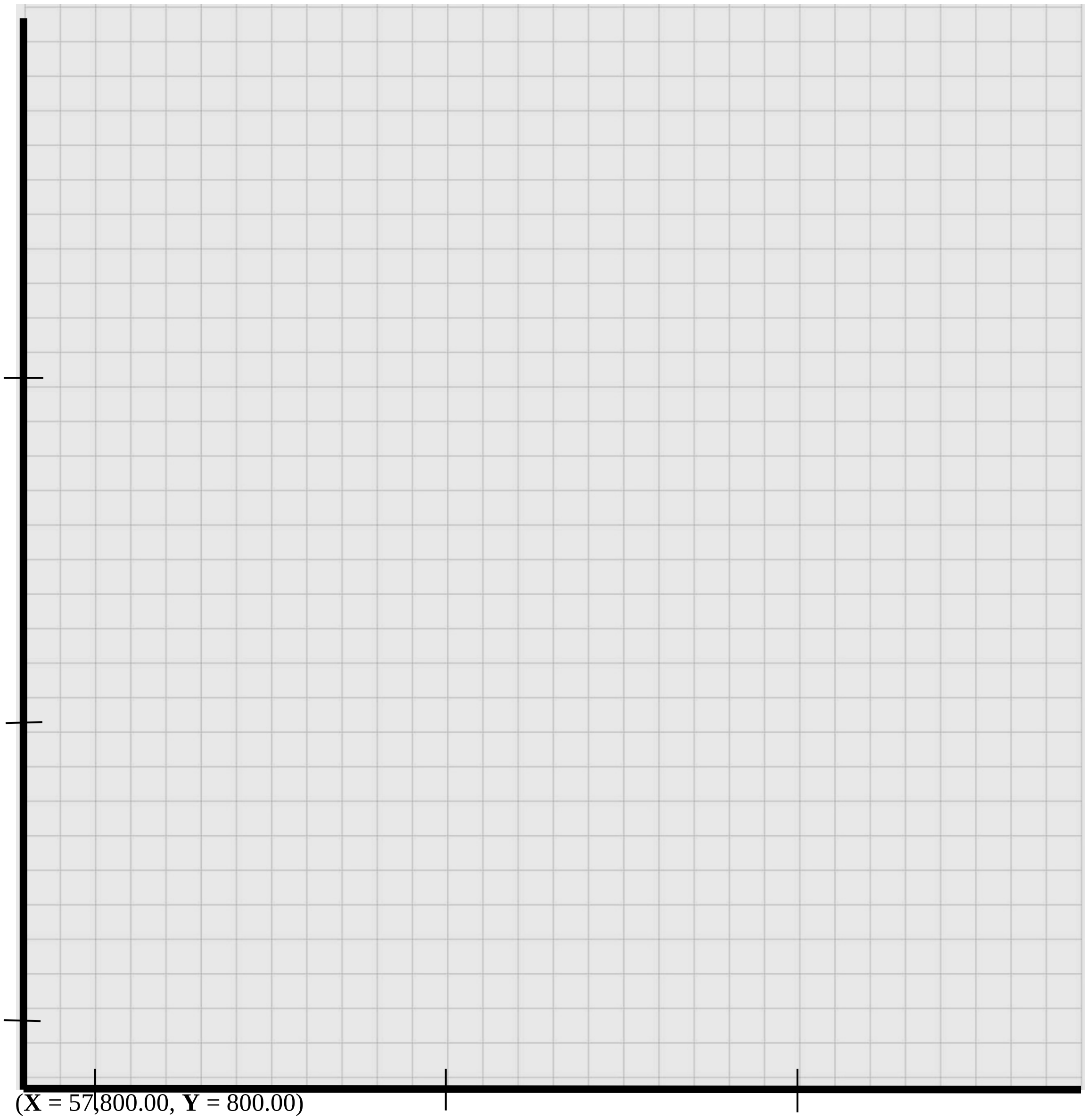
1.) What is the value of the x difference; is it pos. or neg. and why?
x = -21.67 feet. It is negative because it is heading west, or left from the origin.

2.) What is the value of the y difference; is it pos. or neg. and why?
y = 2654.78 feet. It is positive because it is heading North, or up from the origin.

3.) What is the coordinate at the end of the call?
X = (60,000.00 – 21.67) = 59,978.33

Y = (1,000.00 + 2654.78) = 3,654.78

Practice - Coordinates



(**X** = 57,800.00, **Y** = 800.00)

Calculate the coordinates for the following Bearing & Distance starting from the coordinates of **X** = 60,000.00 and **Y** = 1,000.00:

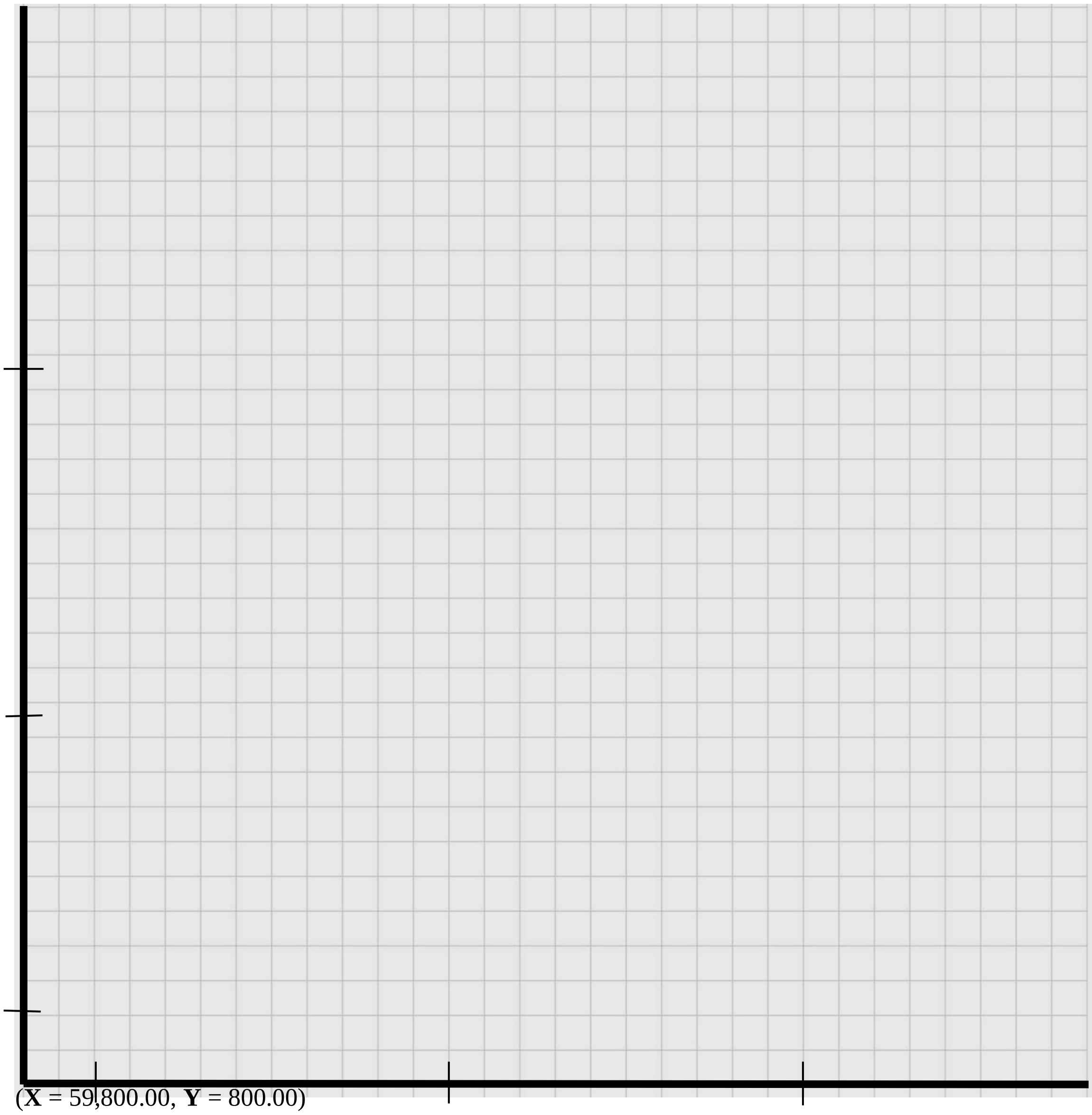
N89-10-33E   2683.68 feet

4.) What is the value of the x difference; is it pos. or neg. and why?

5.) What is the value of the y difference; is it pos. or neg. and why?

6.) What is the coordinate at the end of the call?

Practice - Coordinates



(**X** = 59,800.00, **Y** = 800.00)

Using the coordinates from the Bearing and Distance in questions 6 as a starting point, calculate the coordinates for the following Bearing & Distance:

N45-57-25W 3763.22 feet

7.) What is the value of the x difference; is it pos. or neg. and why?

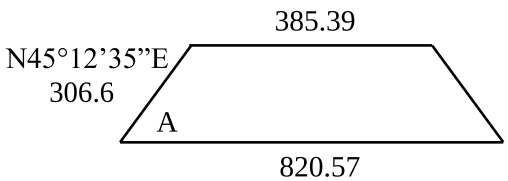
8.) What is the value of the y difference; is it pos. or neg. and why?

9.) What is the coordinate at the end of the call?

Day 3 Practice

Exercise: Solve these Trapezoids.

1.

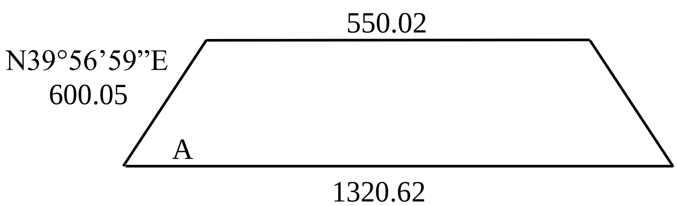


Angle A _______________

Side h _______________

Area ___________________

2. .

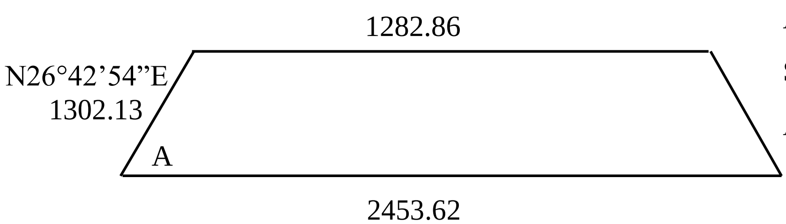


Angle A _______________

Side h _______________

Area ___________________

3. .

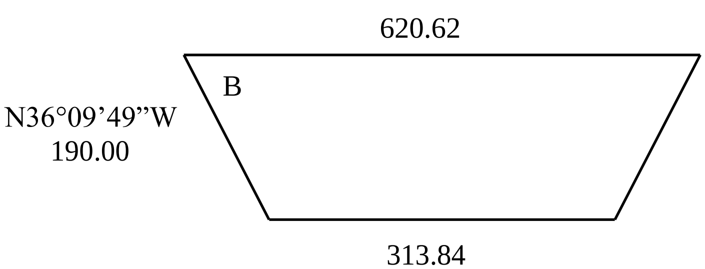


Angle A _______________

Side h _______________

Area ___________________

4. .

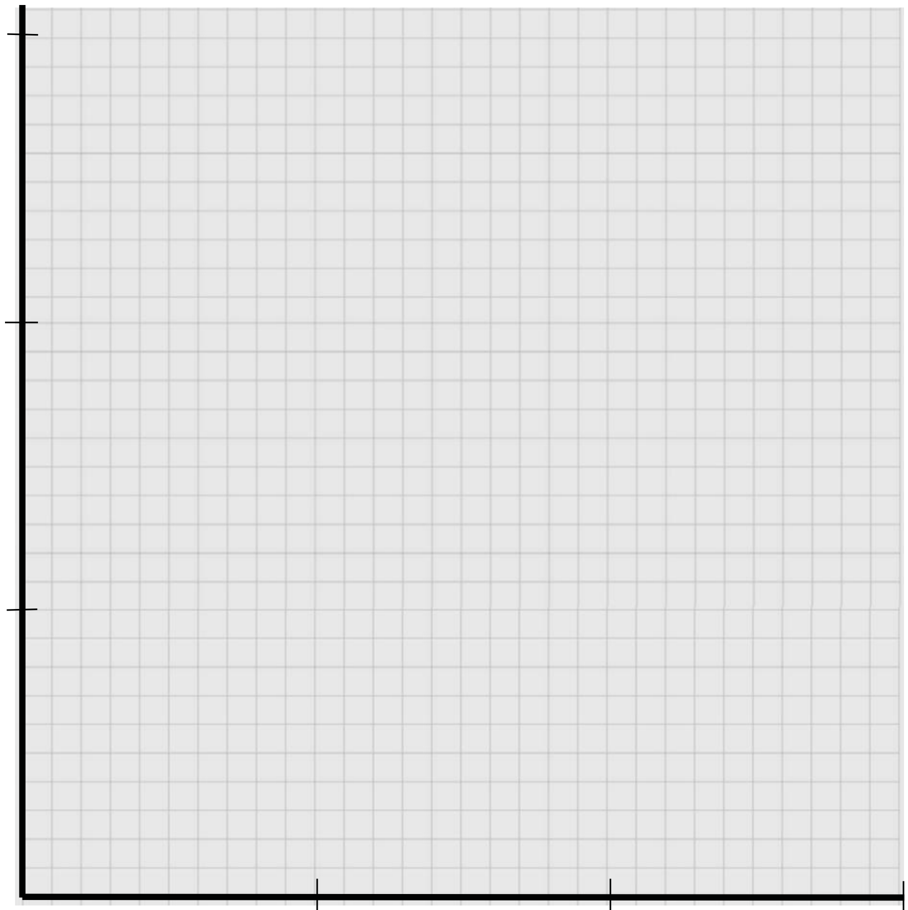


Angle B _______________

Side h _______________

Area ___________________

Day 3 Practice
Coordinates



(X = 7000, Y = 0)

Plot the following coordinates:

Point 1:	Point2:	Point 3:
X = 7200	X=9985.25	X=9985.25
Y = 500	Y= 500	Y=480.59

5.) What is the Angle between Line 1-2 and Line 1-3?

____

6.) What is the Bearing from Point 1 to Point 3?

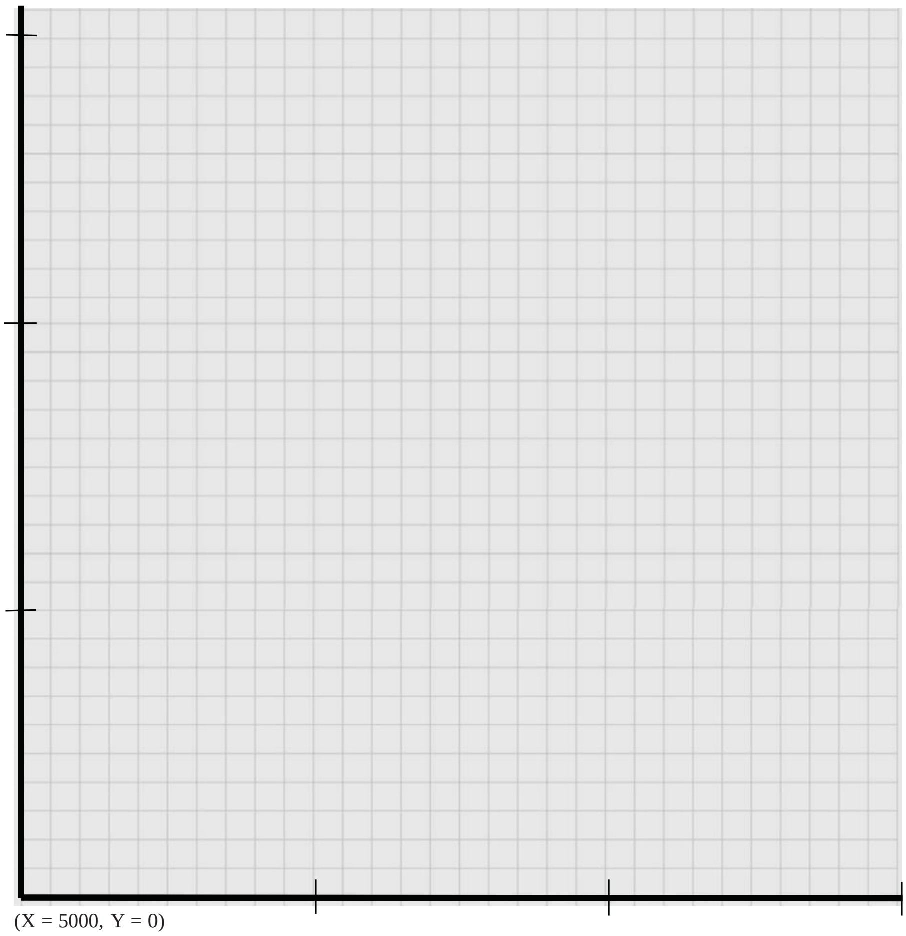
____

7.) What is the Distance from Point 1 to Point 3?

____

Day 3 Practice

Coordinates



Plot the following coordinates:

Point 1:	Point2:	Point 3:
X = 300	X=300	X=326.19
Y = 5300	Y= 7946.81	Y=7946.81

8.) What is the Angle between Line 1-2 and Line 1-3?

____

9.) What is the Bearing from Point 1 to Point 3?

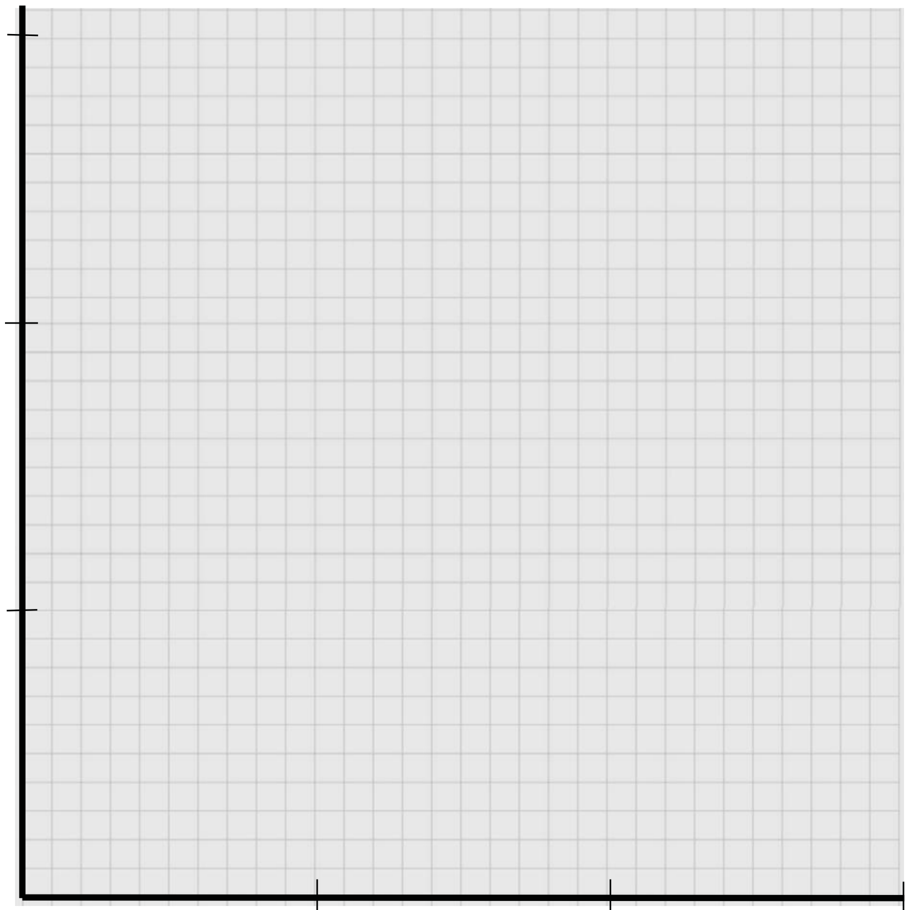
____

10.) What is the Distance from Point 1 to Point 3?

____

Day 3 Practice

Coordinates



(X = 1,400,000, Y =500,000)

Plot the following coordinates:

Point 1:	Point2:	Point 3:
X = 1,402,800	X=1,400,232.83	X=1,402,730.83
Y = 500,500	Y= 500,468.75	Y= 503,162.40

11.) What is the Angle between Line 1-2 and Line 1-3?

____

12.) What is the Angle between Line 1-2 and Line 2-3?

____

13.) What is the Angle between Line 1-3 and Line 3-2?

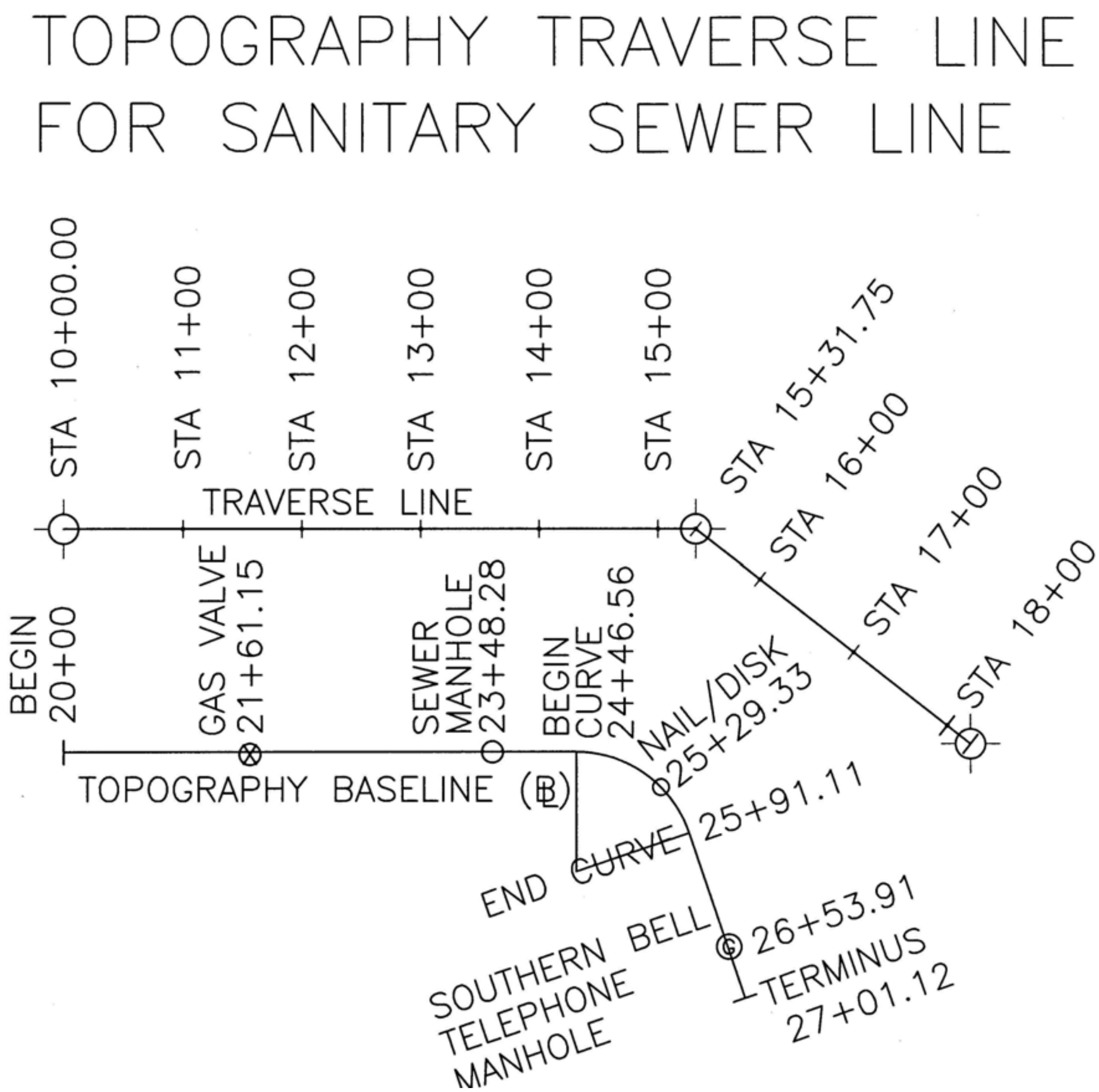
____

14.) What is the distance from Point 2 to Point 3?

____

STATIONING

This is an engineering and surveying form of measurement. It is only a tool to reduce the amount of text data that appear on plans. It usually is not used in boundary surveying. You will come across this type of measurement many times in dealing with property acquired along highways. An example is shown below.



1. What is the distance between?

a) the gas valve and the sewer manhole? ______________________

b) the sewer manhole and the beginning of the curve? ____________

c) the gas valve and the point of beginning? ____________________

d) the Southern Bell telephone manhole and the end of the curve? _____________

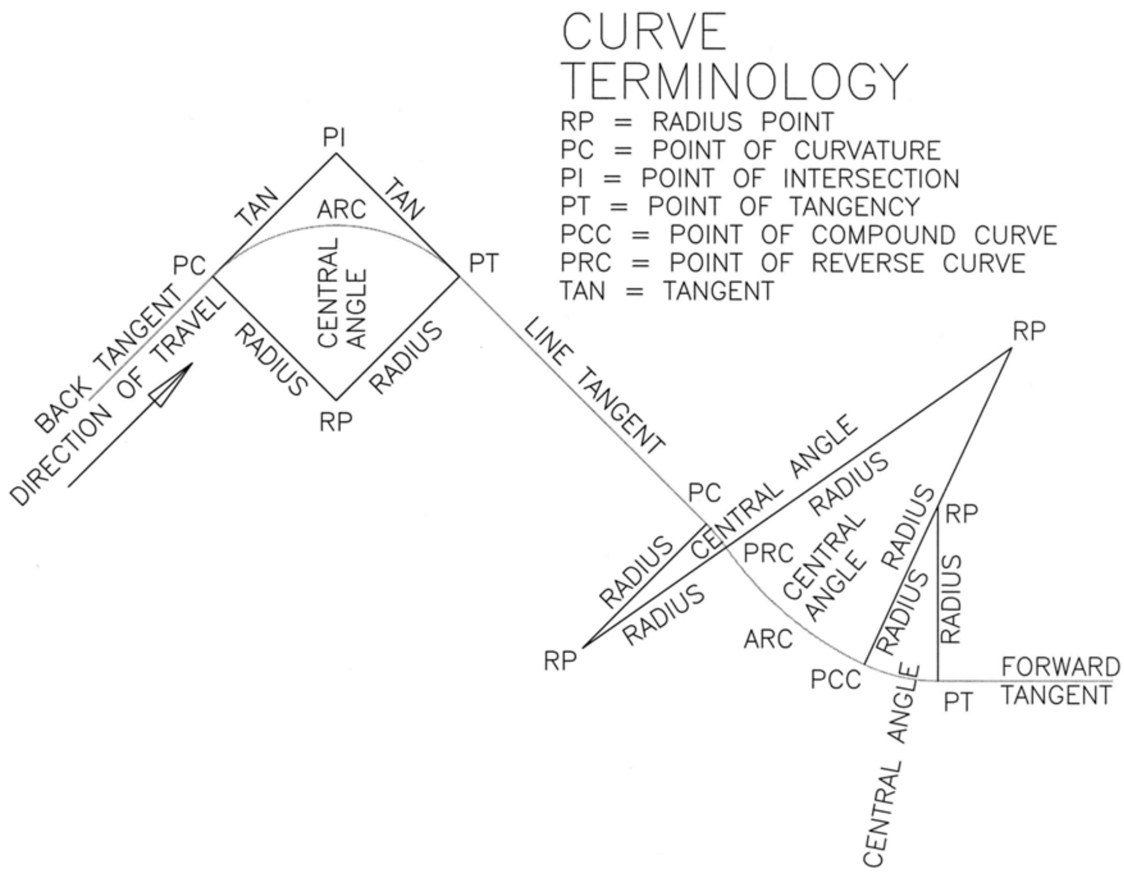
e) the gas valve and the nail and disc? ________________________

2. How long is the curve? _____________________________________

The Circular Curve

In mapping, frequently we find the need to draw or calculate the parts of a circular highway curve or simple curve. To compute the parts of a curve, surveyors used to use the book Length of Circular Arcs for the Radius of One for Each Second of Arc, written by Edward G. Sewoster in 1960 quite extensively, but calculators have now replaced that book. There is special terminology used in the application of curve data. It is shown below and on the next pages.

A typical curve alignment with the geometrical parts of the curve:



Curve Terminology

Arc Length (A) – length of the outside of a portion of the curve.

Back Tangent (BT) – The tangent direction at the start of curve.

Central Angle/Delta Angle (A) – The angle formed by the intersection of the outside radial lines. Delta is actually the name of the Greek letter that symbolizes the central angle and is not used in the rest of this workbook.

Chord (C) – a straight line between two points on an arc.

Degree of Curve (D_A) – central angle that fits a 100 foot arc (arc basis).

Degree of Curve (D_R) – central angle that fits a 100 foot chord on a curve (railroad curve). Seldom used anymore at all.

External (E) – distance from PI to middle of arc.

Forward Tangent (FT) – The tangent direction at the end of the curve.

Long Chord (LC) – straight line distance from PC to PT.

Long Chord Bearing (LCB) – The surveyor's direction of a chord.

Middle Ordinate (M) – length from middle of long chord to the middle of arc.

Point of Compound Curve (PCC) –Where the total deflection of one curve ends and without a tangent line from it, another curve begins and continues radically in the same direction but with a varying length radius.

Point of Curvature (PC) – Where the curve begins to deflect from the tangent.

Point of Intersection (PI) – Where the Back Tangent intersects with a Forward Tangent (tangents are equal in length in a simple curve).

Point of Reverse Curve (PRC) – Where the total deflection of the first curve ends and without a tangent line from it, another curve begins of a varying length radius but leads away in the opposite direction.

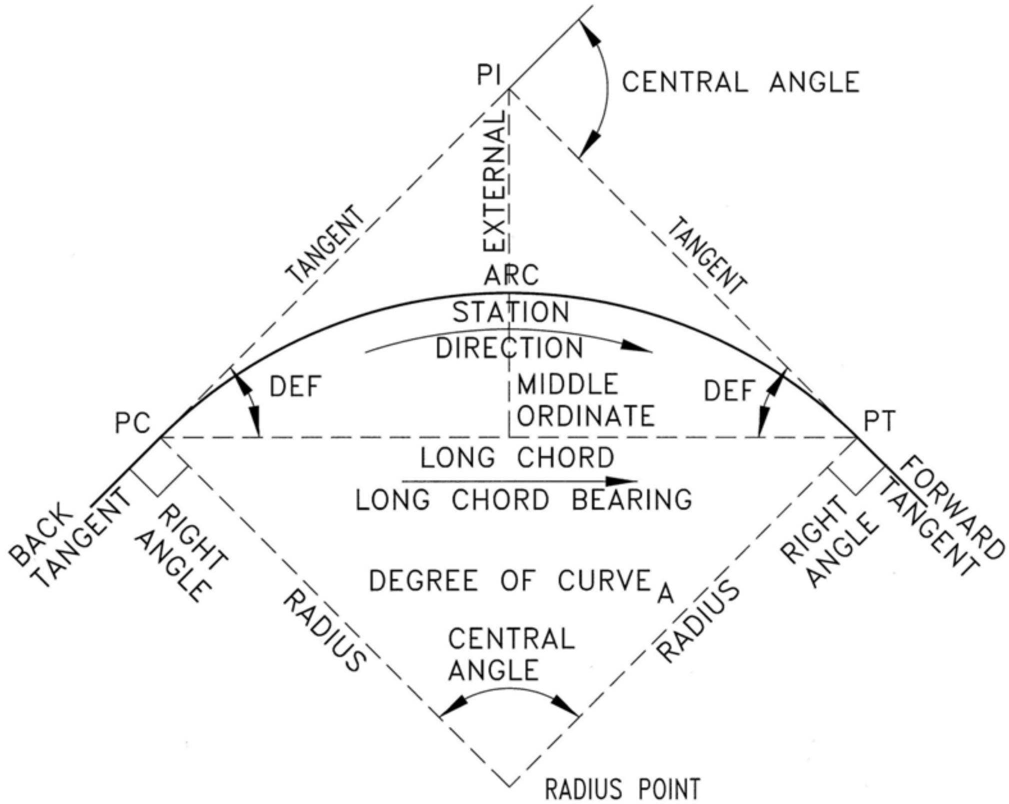
Point of Tangency (PT) – Where the total deflection of the curve ends and the Forward Tangent starts.

Radius (R) – The shortest distance from the center of the curve (circle) to a point on the circumference.

Radius Point (RP) – The point of intersection of the radial lines.

Tangent (Tan or T) – A straight line. The equal lengths (in a simple curve) between the PC and the PI and the PI and the PT.

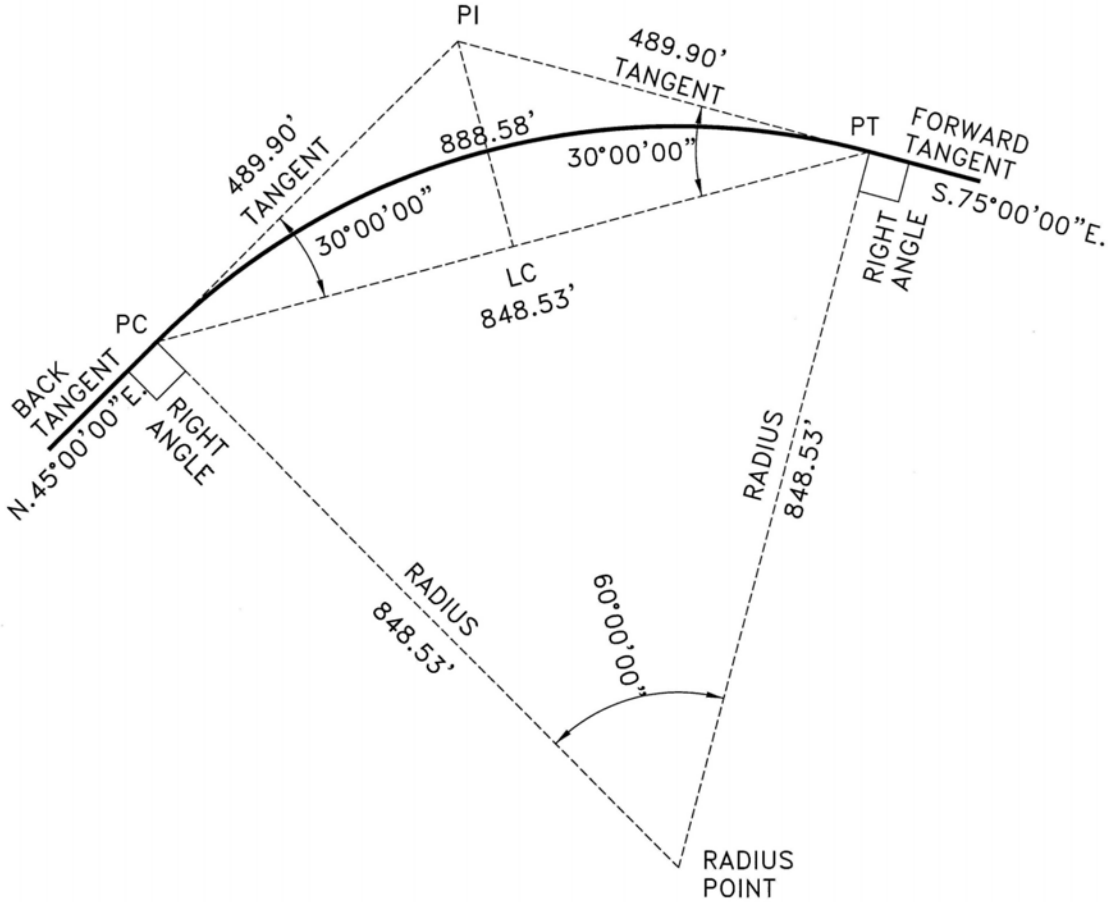
Abbreviations of a Simple Curve



Arc or Arc Length or Length (A or AL or L)	Back Tangent (BT)
Central Angle (A or CA)	Chord (C or ch) – not shown
Deflection Angle (Def or A/2)	Degree of Curve, Arc Basis (D_A)
Degree of Curve, Railroad Basis (D_R) – not shown	External (E)
Forward Tangent (FT)	Line Tangent (LT)
Long Chord (LC)	Long Chord Bearing (LCB) – not shown
Middle Ordinate (M)	Point of Compound Curve (PCC)–n/s
Point of Curvature (PC)	Point of Intersection (PI)
Point of Reverse Curve (PRC) – not shown	Point of Tangency (PT)
Radius (R)	Radius Point (RP)
Station Direction (SD)	Tangent (T or Tan)

The first abbreviation shown is the preferred one.

Typical Curve



R = 848.53' (radius)

∆ = 60º00'00" (central angle)

A = 888.58' (arc length)

LC = 848.53' (long chord or chord)

Def ∆ = 30º00'00" (one-half of central angle)

M = 131.68' (What does this mean? Where is this?)

E = 131.27' (What does this mean? Where is this?)

T = 489.90' (tangent)

BT (Back Tangent) = N.45°00'00"E. (Where is this?)

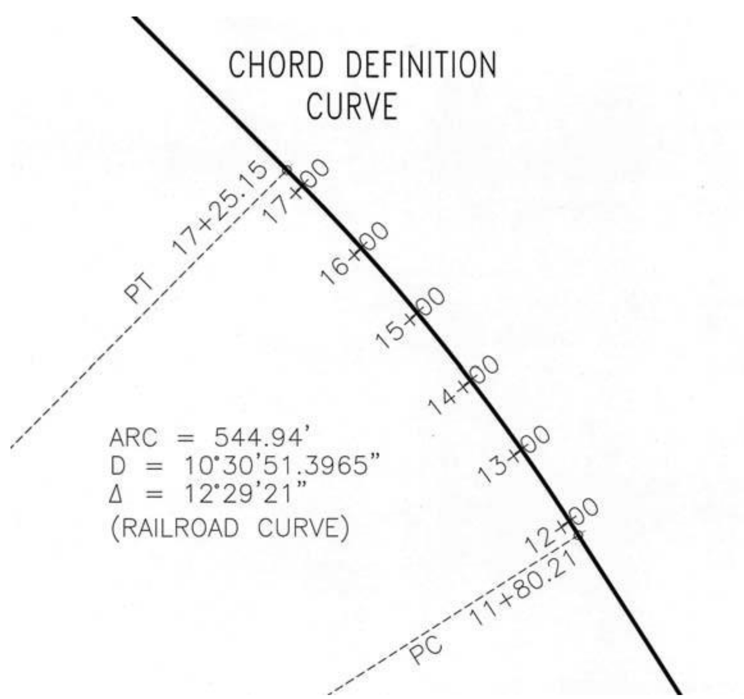
FT (Forward Tangent) = S.75°00'00"E. (Where is this?)

Curve Computations

The sharpness of a curve's turn may be designated by the radius: as the radius shortens, the curve sharpens. As the radius lengthens, the curve gets straighter (flatter). The value of the radius is important to the calculations of a curve. The sharpness of a curve is expressed by the term "Degree of Curve". There are two types of curves: a railroad curve and a highway curve.

Railroad Curve (D_R)

The railroad curve or Curve by Chord Definition is a curve that is measured along various short chords from the PC to PT. The Railroad Degree of Curve is expressed thus: the Degree of Curve of a central angle of 01°00‘00" when subtended by a 100 foot chord. Or as otherwise expressed: a central angle of 01°00'00" with a long chord of 100 feet.

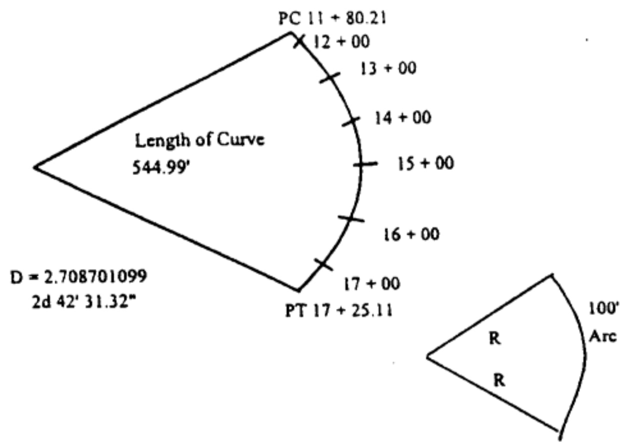


It is computed as Radius = 50'/Sin (01°00'00" / 2) or 5729.65067401 feet or for normal terms: 5729.6507 feet (railroad definition, chord basis). This curve is used very seldom anymore. It was used when railroads were being built like crazy (up thru the 1920's), but no more.

Highway Curve

The Highway Curve or Curve by Arc Definition is a curve that is measured along the arc from the PC to the PT. The Degree of Curve (D_A) defined as the central angle that is formed when subtended by a 100 foot arc, or as otherwise expressed: a central angle of 01°00’00” having an arc length of 100 feet.

Arc Definition



Or as Radius = 100’ / (n/180) or 5729.57877856 feet or for normal terms 5729.5788 feet (arc or highway definition). Not much difference when compared with the railroad curve, but enough to mess up the curve computations. Be very, very sure of what type you are working with before you do a bunch of computations with the wrong one.

Since highway curves are the curves most commonly plotted in Property Appraiser’s Offices and surveyors typical use nothing but highway curves, we will deal with that type from now on in this course. If you have a highway that parallels (follows) a railroad right-of-way, look at any curves in it with caution. Not all highway curves are of the highway definition. In fact, not all railroad curves are railroad definition. Some are highway curve radius length. If you are having problems, check using both radii and see which curve really works.

Pi is a Greek letter which represents the ratio of the circumference of a circle to its diameter. To the radius it equals the curvature of half a circle. It is equal to 3.14159 26535 89793 23846 for half a circle.

Curve Formulas

(The central angle is always processed in decimal degrees.)

ARC DEFINITION:

D_c (degree of curve, arc definition or highway definition) = 5729.5788 / R

R = 5729.5788 / D_c

(Notice that the above formulas are related.)

Arc Length = 100 x (3 / degree of curve) or Arc = $\underline{100 \ x \ \ 3}/D_c$

T = R * tan (½ A)

(Long) Chord = 2R * sin (½ A)

M = R – (R cos (½ A))

E = (R / cos ½ A) - R

SECTOR AREA = $(nR^2) * (\Delta / 360)$ (see next page)

SEGMENT AREA = Sector Area – $\frac{R^2}{2} * \sin \Delta$ (see next page)

FILLET = R * T – $(nR^2) * (\Delta / 360)$ (see next page)

PC = PI Station - T (tangent length)

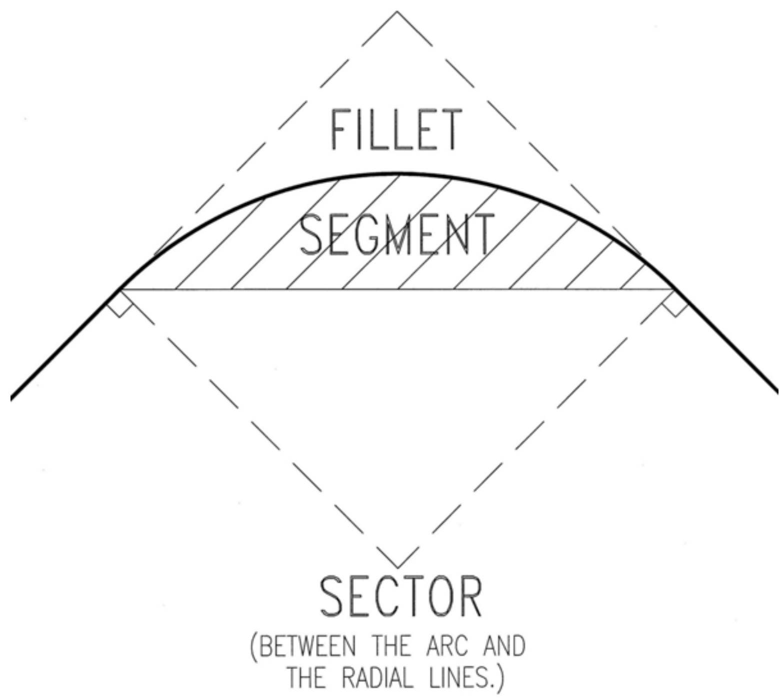
PT = PC Station + Arc length

The Central Angle can be found by the use of any formula that contains the Central Angle as a part of the formula, just switch the parts around to solve for the Central Angle. For example: the tangent formula method. The original formula is T = R * tan (1/2 3). To change this around to where one solves for the Central Angle: $\frac{\text{Tan } 3}{2} = \underline{T / R.}$

CURVE AREAS

The areas within a curve can be calculated as well. How you compute the area of a parcel depends on how you compute the area of the curve or curves that you need. If you go into the radius point for a curve, you need the Sector area (between the radius lines and the arc) to help you compute the area of the parcel. If you use the long chord (typical), you need the Segment area (between the long chord and the arc). If you use the tangent lines, you need the Fillet area (between the tangent lines and the arc) to compute the overall parcel.

Not knowing which method you will use, we will give you the formulae for all three areas and you can choose.



Sectors:
SECTOR = $(n R^2) * (\Delta / 360)$

Segments:
SEGMENT = Sector – $\frac{R^2}{2} \sin \Delta$

Fillets:
FILLET = $R * T – (nR^2 * (\Delta / 360))$
or
FILLET = R * T - Sector

As you can see, the SECTOR Area is of primary importance. Once you know how to compute that figure the rest falls into place.

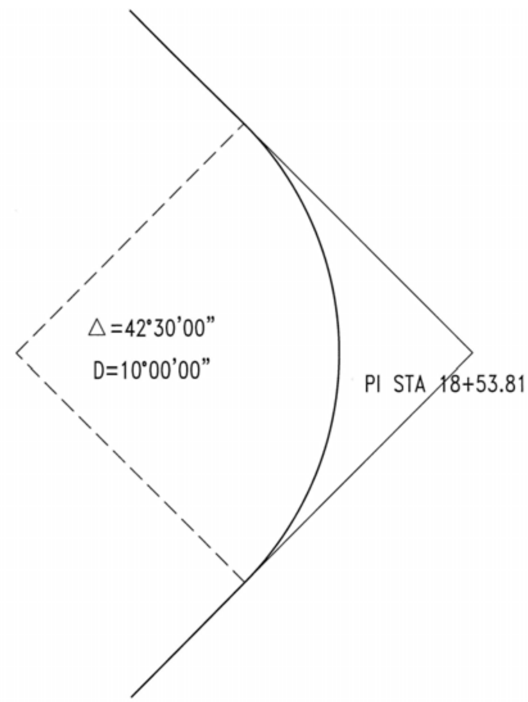
Curve Example, Arc definition:

GIVEN:

Delta Angle = 42º 30’ 00”

Degree of Curve = 10º 00’ 00”

PI Station = 18 + 53.81



Find:

Radius = ______________________________

Tangent = ______________________________

PC Station = ______________________________

PT Station = ______________________________

Arc Length =

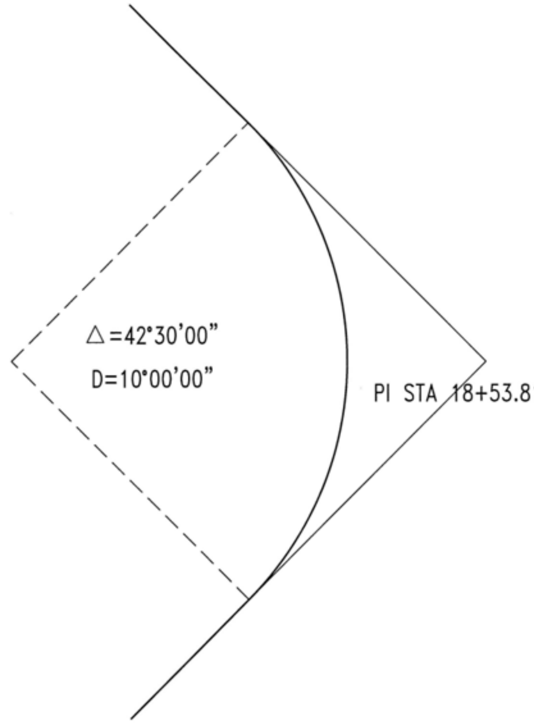
Deflection (Def) = ______________________________

Long Chord = ______________________________

Area of Sector = ______________________________

Area of Segment = ______________________________

Area of Fillet = ______________________________



ARC DEFINITION CURVE

Radius (R): (The FIRST figured! Why?)

R = 5729.5788/D (formula)
R = 5729.5788/10°00'00" (plug in the numbers)
R = 5729.5788/10.000000 (convert to decimal degrees)
R = 572.96' (Divide and get the answer)

Tangent (T):

T = R * tan (½ central angle) (the formula)
T = 572.96 * tan (42º30'00"/2) (plug in the numbers)
T = 572.96 * tan 21°15'00" (central angle / 2)
T = 572.96 * tan 21.250000 (get the decimal delta)
T = 572.96 * 0.388878732 (get the tangent figure)
T = 222.81 (multiply and get answer)

PC Station (PC): (For the PC Station, use the tangent.)

PC Station = PI Station – T (the formula)
PC Station = 18+53.81 – 222.81 (plug in the numbers and subtract)
PC Station = 16+31.00 (answer)

Arc Length (A):

Arc = 100 * (central angle/D) or Arc = (central angle * R * п) / 360 (the formula)
Arc = 100 * (42º30'00" / 10°00'00") (plug in the numbers)
Arc = 100 * (42.5/10.000000) (create decimal central angle and decimal degree of curve)
Arc = 100 * 4.25 (divide right side first)
Arc = 425.00' (multiply and get answer)

PT Station (PT): (We do this after figuring the PC Station and Arc Length.)

PT Station = PC Station + Arc (formula)
PT Station = 16+31.00 + 425.00 (plug in and add)
PT Station = 20+56.00 (answer)

Deflection (Def):

Def = central angle/2 (formula)
Def = 42°30'00" / 2 (plug in the numbers)
Def = 42.5 / 2 (change to decimal degrees)
Def = 21.25 (divide the decimal degrees)
Def = 21°15'00" (convert back to degrees, minutes and seconds and get answer)

Long Chord:

LC = 2R * sin ½ central angle (formula)
LC = (2 * 572.96) * sin 42°30'00 / 2 (plug in the numbers)
LC = (2 * 572.96) * 0.362438038 (get sin of half central angle converted to decimal degrees)
LC = 1145.92 * 0.362438038 (convert radius)
LC = 415.32' (multiply and get answer)

Sector Area:

Sector = (Δ / 360) * (πR^2) (the formula) or Sector = (πR^{2}Δ) / 360

Sector = (42°30'00" / 360) * (3.14159265 * 572.96^2) (plug in the numbers)

Sector = (42.5 / 360) * (3.14159265 * 572.96^2) (convert to decimal degrees and reduce)

Sector = (0.11805556) * (1,031,331.9688) (multiply overall)

Sector = 121,754.4685 square feet (answer) or 121,754 square feet normally.

Segment Area:

Segment = Sector – $\frac{R^2}{2}$ sin Δ (the formula)

Segment = 121,754.4685 – $\frac{572.96^2}{2}$ sin 42°30'00" (plug in numbers, see Sector)

Segment = 121,754.4685 – $\frac{328,283.1616}{2}$ sin 42.5° (square and convert to decimal degrees)

Segment = 121,754.4685 – $\frac{328,283.1616}{2}$ * 0.67559021 (get the sin, divide and multiply)

Segment = 121,754.4685 – 110,892.4447 (subtract overall)

Segment = 10,862.0238 square feet (answer) or 10,862 square feet normally.

Fillet Area:

Fillet = R * T – (Δ/360) * (πR^2) (the formula)

Fillet = 572.96 * 222.81 – (121,754.4685) (plug in the numbers, see Sector)

Fillet = 127,661.2176 – (121,754.4685) (multiply left)

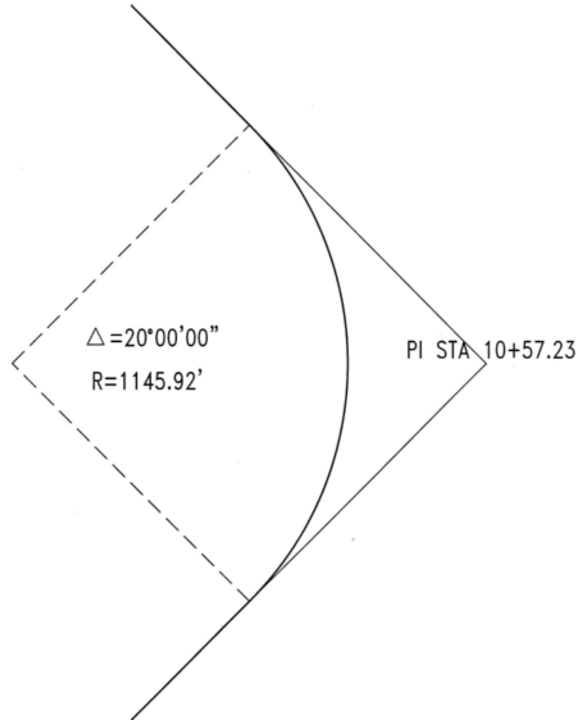
Fillet = 127,661.2176 – 121,754.4685 (subtract overall)

Fillet = 5,906.7491 square feet (answer) or 5,907 square feet normally.

Again, as you can see, the Sector Area is of primary importance. You must have its operations down pat before you can compute the remaining portions with any degree of confidence.

Curve Exercise
Given:

Radius	=	1145.92'
Delta	=	20º 00' 00"
PI Station	=	10+57.23



Find:

Degree of Curve = ____________________________

Arc Length = ____________________________

Tangent = ____________________________

Long Chord = ____________________________

Sector Area = ____________________________

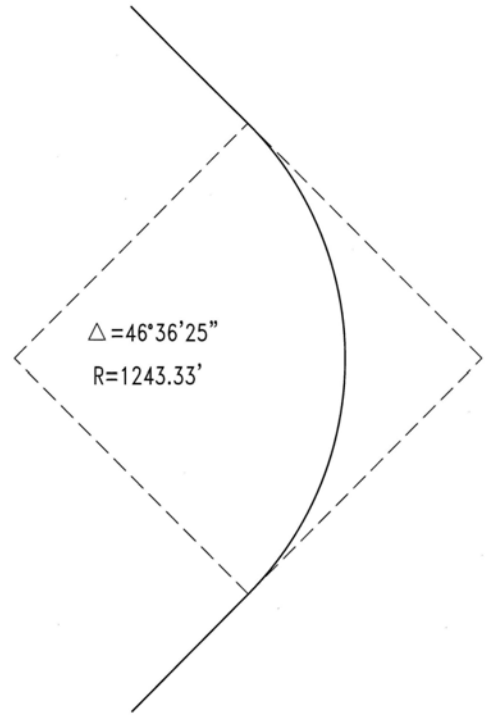
PC Station = ____________________________

Curve Exercise

Given:

Radius = 1243.33'

Delta = 46º 36' 25"



Find:

Degree of Curve = ______________________

Arc Length = ______________________

Tangent = __________________

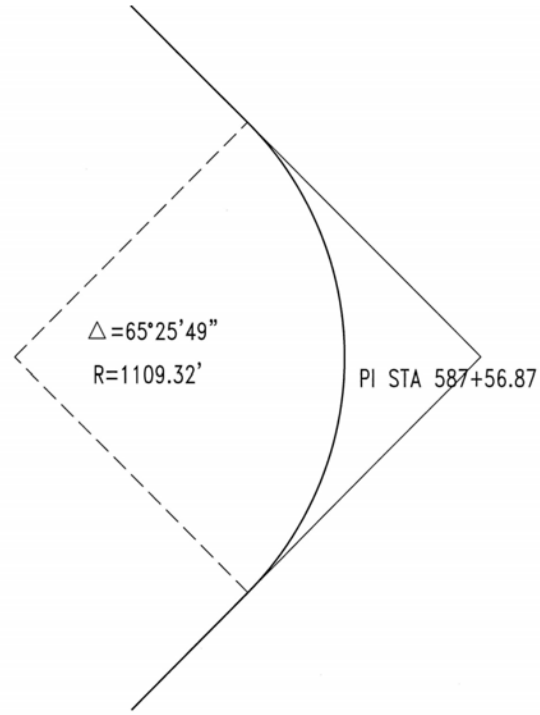
Long Chord = __________________

Fillet Area = __________________

Curve Exercise

Given:

Radius	=	1109.32
Delta	=	65º 25' 49"
PI Station	=	587+56.87



Find:

Degree of Curve = __________________

Arc Length = __________________

Tangent = __________________

Long Chord = __________________

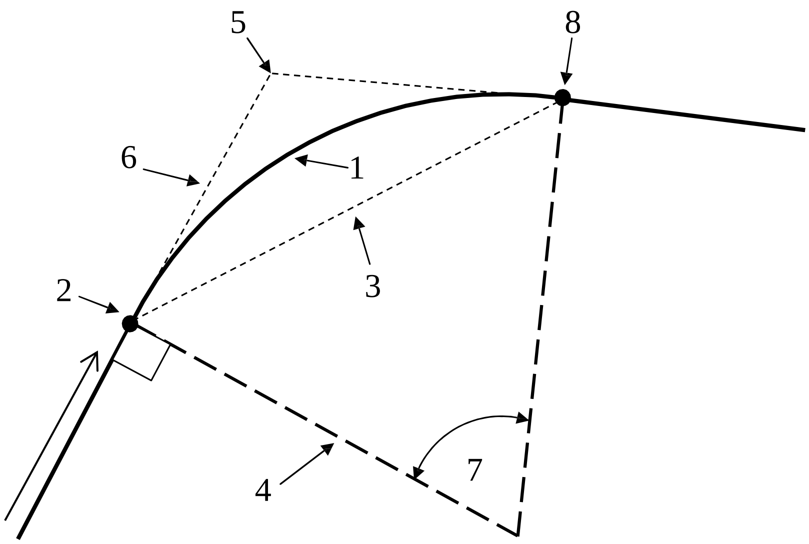
Segment Area = __________________

PT Station = __________________

Day 4 Practice

Curve Exercise: Match the elements of the curve and write down the numbers that correspond

1.



Radius = _______

Tangent = _______

PC = _______

PT = _______

Central Angle = ______

Arc Length = _______

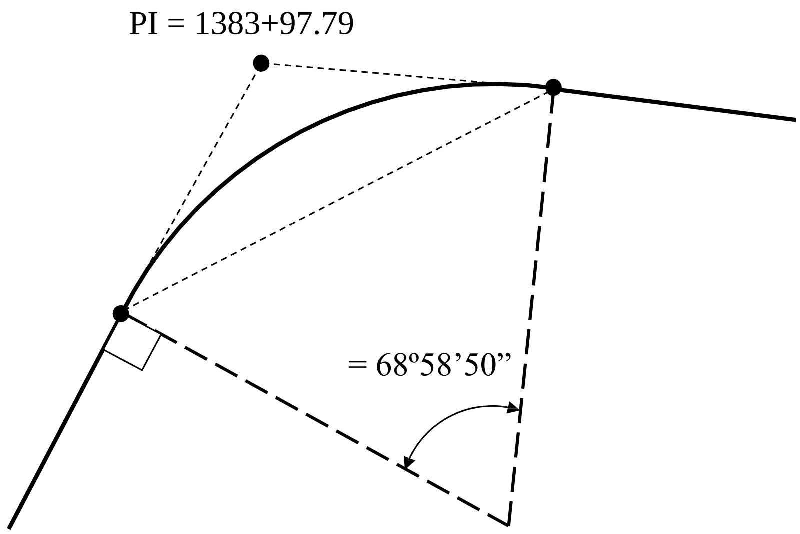
PI = _______

Long Chord = _______

Day 4 Practice

Curve Exercise: Arc definition:

2.



GIVEN:
Delta Angle = 68º58'50"

Degree of Curve = 02º00'00"

PI Station = 1383+97.79

Find:

Radius = ____________________________________

Tangent = ___________________________________

PC Station = _________________________________

PT Station = __________________________________

Arc Length = _________________________________

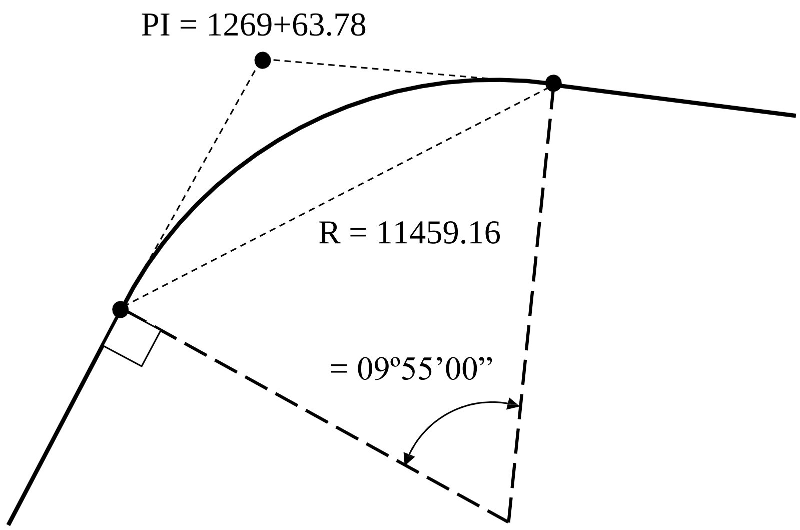
Deflection (Def) = ____________________________

Long Chord = _________________________________

Day 4 Practice

Curve Exercise: Arc definition:

3.



GIVEN:
Radius = 11459.16

Delta = 09º55’00”

PI Station = 1269+63.78

Find:

Degree of Curve = ______________________________

Arc Length = __________________________________

Tangent = _______________________________________

Long Chord = __________________________________

Sector Area = __________________________________

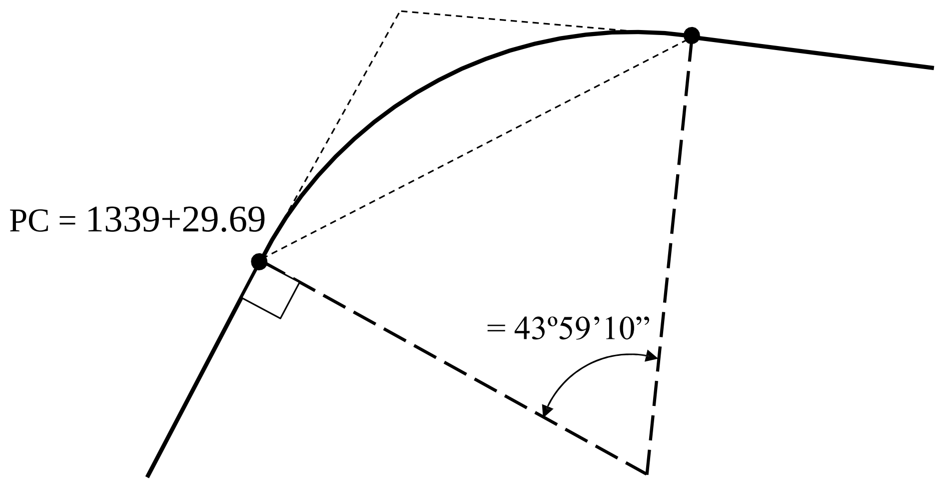
PC Station = ___________________________________

PT Station = ____________________________________

Day 4 Practice

Curve Exercise: Arc definition:

4.



GIVEN:
Tangent = 771.36

Delta = 43º59’10”

Tangent Bearing (Curve Right) = N03-20-34E

PC Station = 1339+29.69

Find:
Degree of Curve = ____________________________

Arc Length = _________________________________

Long Chord = ________________________________

Radius = _____________________________________

Chord Bearing = ______________________________

PT Station = _________________________________

PI Station = __________________________________

AREA FORMULAS

Rectangle	Length * Width = Area
	Area/Width = Length
	Area/Length = Width
Triangle (Right)	1/2*(Height * Base) = Area
	or
	(Height * Base)/2 = Area
	2 * Area/Base = Height
	2 * Area/Height = Base
Circle	n * Radius^2 = Area
	Area/n = Radius
Square	Length * Width = Area
Parallelogram	Height * Base = Area
Trapezoid	Height * (Top Line + Bottom Line/2) = Area (top and bottom line are parallel)

PERIMETER

Object:	Formulae:
Rectangle	a+b+c+d = Perimeter
Triangle	a+b+c = Perimeter
Square	a+b+c+d = Perimeter
Parallelogram	a+b+c+d = Perimeter
Trapezoid	a+b+c+d = Perimeter
Circle	2R*n or d*n = Circumference

Types of Triangles:

A plane triangle has three sides and 180 degrees in angle size. It may be classified by how many of its sides are of equal length. Or, it may be classified by what kind of angles it has.

An **equilateral triangle** is a triangle where all three sides are equal and it is also equiangular, meaning each angle measures 60 degrees.

In a **right triangle**, one of the angles is a right angle, meaning it measures 90 degrees. A right triangle may also be an isosceles or scalene triangle.

In an **obtuse triangle** one of the angles is greater than 90 degrees, and may be an isosceles or scalene triangle.

In an **acute triangle** all angles are less than 90 degrees. An acute triangle may be equilateral, isosceles or scalene.

In an **isosceles triangle** two sides are the same length. An isosceles triangle may be right, obtuse, or acute.

In a **scalene triangle** none of the sides are the same length. A scalene triangle may be right, obtuse or acute.

GLOSSARY

Abscissa – The x-coordinate (easterly) in Cartesian Coordinates.

Acute Angle – An angle smaller than 90 degrees.

Acute Triangle – A triangle wherein each of the three angles is smaller than 90 degrees, but still has a total of 180 degrees.

Adjacent Angles – Two angles are adjacent if they share the same vertex and have one side in common.

Angle – The union of two lines with a common endpoint.

Angle of Depression – The angle of depression for an object below your line of sight is the angle whose vertex is at your position, with one side being the horizontal line in the same direction as the object and the other side being the line from your eye passing through the object. Not in test.

Angle of Elevation – The angle of elevation for an object above or below your line of sight is the angle whose vertex is at your position, with one side being a horizontal line from your eye passing through the object. Not in test.

Arc –A set of points on a circle that lie in the interior of a particular central angle.

Axis – The y-axis in Cartesian Coordinates (northerly).

Cartesian Coordinate – A system whereby points on a plane are identified by an ordered pair of numbers, representing the distances to two perpendicular axes. The horizontal axis is usually called the x-axis, and the vertical axis is usually called the y-axis.

Central angle – An angle that has its vertex at the center of arc.

Chord – A line segment that connects two points on a curve.

Circle – Set of points in a plane that are all a fixed distance from a given point.

GLOSSARY Continued:

Circumference – It is the total distance around the outer edge of the complete circle. The circumference of a complete circle is $2\pi r$, where r is the radius and π equals pi (also: $d*\pi$ where d = diameter).

Compass – A device consisting of two adjustable legs, used for drawing circles and measuring off distance intervals. Also a direction finder (used to determine or set direction used in surveying).

Complementary Angles – Two angles are complementary if the sum of their measures equals 90 degrees.

Decagon – A polygon with 10 sides. A <u>regular</u> decagon has 10 equal sides and 10 angles, each measuring 144 degrees.

Degree – A unit of measurement for angles. One degree is equal to $1/360^{th}$ of a full rotation (circle).

Diameter – The length of a line segment adjoining two points on the circle and passing through the center ($d*\pi$ = circumference).

Equilateral Triangle – A triangle with three equal sides.

Euclidian Geometry –The geometry based on the postulates of Euclid, who lived in Alexandria, Egypt in 300 B.C.

Geometry – The study of shape and size. Normal geometry is based on the work of Euclid.

Hexagon – A six sided polygon. The sum of the angles in a hexagon is 720 degrees. <u>Regular</u> hexagons have six equal sides and six equal angles of 120 degrees each.

Hypotenuse – The side of the right triangle that is opposite the right angle. It is the longest of the three sides of the triangle. It is commonly shown as “c”.

Isosceles triangle – An isosceles triangle has two equal sides.

GLOSSARY Continued:

Minute – A unit of measure for angle equal to 1/60 of a degree.

Oblique angle – An angle that is not a right angle.

Oblique triangle – A triangle that is not a right triangle.

Obtuse angle – An angle larger than 90 degrees and smaller than 180 degrees.

Obtuse triangle – A triangle containing at least one obtuse angle.

Octagon – An eight sided polygon.

Ordinate –Another name for the **y** coordinate.

Origin – The origin is the point (0, 0) in Cartesian Coordinates.

Parallelogram – A quadrilateral with opposite sides parallel.

Pentagon – A five sided polygon. The sum of the angles in a pentagon is 540 degrees. A regular pentagon has all five sides equal, and each of the five angles is equal to 108 degrees each.

Perimeter – The sum of the lengths of all the sides.

Pi – the ratio between the circumference of a circle and its diameter. The decimal approximation of pi is 3.1415926536. The symbol for pi is n (a Greek letter).

Polar Coordinates – Any line in a plane can be identified by its distance from the origin and its angle of inclination.

Polygon – A union of several line segments that are joined end to end so as to completely enclose an area.

Protractor – A device for measuring the size of angles.

Quadrilateral – A four sided polygon.

GLOSSARY Continued:

Radius – The shortness distance from the center of the circle to a point on the circle.

Rectangle – A quadrilateral with four 90 degree angles. The opposite sides of the rectangle are parallel.

Right angle – An angle that measures 90 degrees.

Right triangle – A triangle that has one angle which measures 90 degrees.

Scalene triangle – A triangle which has no sides equal.

Second – A unit of measure of an angle equal to 1/60 of a minute or 1/3600 of a degree.

Trapezoid – A quadrilateral that has just two sides parallel.

Triangle – A three sided polygon totaling 180 degrees.

ANSWERS

The Addition of Angular Measure……………………………………………..…..Pg. 18

1. 81° 02’ 51”
2. 77° 52’ 07”
3. 57° 00’ 35”
4. 125° 28’ 06”
5. 135° 24’ 36”

The Subtraction of Angular Measure………………………………………………..Pg. 23

1. 39° 12’ 49”
2. 32° 31’ 53”
3. 26° 16’ 12”
4. 57° 39’ 35”
5. 63° 33’ 37”

Multiplication and Division of Angular Measure………………………………….Pg. 26

1. 273° 34’ 54”
2. 279° 47’ 56”
3. 805° 42’ 27”
4. 40° 06’ 05”
5. 90° 11’ 09”
6. 90° 07’ 09”

Converting Angular Measurement to Decimal Degrees…………………………Pg. 27

1. 89.461944445
2. 27.580833333
3. 35.899444444
4. 47.889166667
5. 125.359444444
6. 01.395833333
7. 32.542222222

Decimal of Degrees to Degrees-Minutes-Seconds…………………………………..Pg. 28

1. 25° 14’ 04” -4
2. 87° 59’ 14”
3. 34° 34’ 04” -4
4. 83° 00’ 01” -2
5. 125° 40’ 24”
6. 02° 20’ 59” -4
7. 149° 07’ 26”

Bearings and Angles – Conversion of Bearings to Angles………………… Pgs 32 & 33

(Note: you may have solved a differently, is ok, as long as you attained the correct answer.)

1. Answer: **South71º29'29'East**

$$\begin{array}{rrrr} & 179^\circ & 59' & 60'' \\ - & 88^\circ & 15' & 21'' \\ \hline & 91^\circ & 44' & 39'' \end{array}$$

$$\begin{array}{rrrr} & 91^\circ & 44' & 39'' \\ - & 20^\circ & 15' & 10'' \\ \hline & \mathbf{71^\circ} & \mathbf{29'} & \mathbf{29''} \end{array}$$

2. Answer: **North65º11'41'East**

$$\begin{array}{rrrr} & 110^\circ & 21' & 56'' \\ - & 45^\circ & 10' & 15'' \\ \hline & \mathbf{65^\circ} & \mathbf{11'} & \mathbf{41''} \end{array}$$

3. Answer: **North60º00'00'East**

$$\begin{array}{rrrr} & 180^\circ & 00' & 00'' \\ - & 120^\circ & 00' & 00'' \\ \hline & \mathbf{60^\circ} & \mathbf{00'} & \mathbf{00''} \end{array}$$

4. Answer: **South34º59'50'West**

$$\begin{array}{rrrr} & 93^\circ & 59' & 60'' \\ - & 59^\circ & 00' & 10'' \\ \hline & \mathbf{34^\circ} & \mathbf{59'} & \mathbf{50''} \end{array}$$

5. **Ace Road**: Answer: **North28º29'45'West**

$$\begin{array}{rrrr} & 180^\circ & 00' & 00'' \\ - & 130^\circ & 00' & 00'' \\ \hline & \mathbf{50^\circ} & \mathbf{00'} & \mathbf{00''} \end{array}$$

$$\begin{array}{rrrr} & 78^\circ & 29' & 45'' \\ - & 50^\circ & 00' & 00'' \\ \hline & \mathbf{28^\circ} & \mathbf{29'} & \mathbf{45''} \end{array}$$

6. **Bummy Road**: Answer: **North56º30'15'East**

$$\begin{array}{rrrr} & 134^\circ & 59' & 60'' \\ - & 78^\circ & 29' & 45'' \\ \hline & \mathbf{56^\circ} & \mathbf{30'} & \mathbf{15''} \end{array}$$

Bearings and Angles – Conversion of Angles to Bearings……………………… Pages 34 & 35

(Note: you may have solved a differently, is ok, as long as you attained the correct answer.)

1. . Answer: **105º40'19"**

	51º	32'	45"
+	22º	46'	56"
	74º	19'	41"

	179º	59'	60"
-	74º	19'	41"
	105º	**40'**	**19"**

2. Answer: **138º52'43"**

	31º	32'	23"
+	09º	34'	54"
	41º	07'	17"

	179º	59'	60"
-	41º	07'	17"
	138º	**52'**	**43"**

3. Answer: **83º20'43"**

	88º	43'	57"
-	05º	23'	14"
	83º	**20'**	**43"**

4. Answer:

- **45º**
- **90º**
- **45º**

1. 1.
2. 2.
3. 3.

Formulas

Right Triangles

Pythagorean Theorem

$a^2 + b^2 = c^2$

$c^2 - a^2 = b^2$

$c^2 - b^2 = a^2$

Solve for hypotenuse:

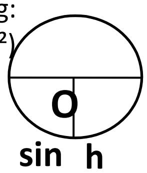
$c = \sqrt{(a^2 + b^2)}$

Solve for leg:

$a = \sqrt{(c^2 - b^2)}$

Sine

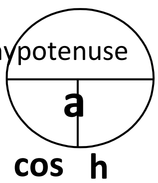
Sine = opposite side ÷ hypotenuse



sine functions:
sine of an angle = opposite side ÷ hypotenuse
opp. side = hypotenuse x sine of opp. angle
hypotenuse = opp. side ÷ sine of opp. angle

Cosine

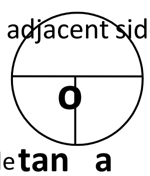
Cosine = adjacent side ÷ hypotenuse



cosine functions:
cosine of an angle = adjacent side ÷ hypotenuse
adj. side = hypotenuse x cosine of adj. angle
hypotenuse = adj. side ÷ cosine of adj.angle

Tangent

Tangent = opposite side ÷ adjacent side



tangent functions:
tangent of an angle = opposite side ÷ adjacent side
opposite side = adjacent side x tangent of an angle
adjacent side = opposite side ÷ tangent of an angle

Coordinate System

Bearing formula:

$\mathbf{(x_1 - x_2) \div (y_1 - y_2) = \tan\theta}$ **(x₁ - x₂) ÷ (y₁ - y₂) = tan theta**

Distance formula:

$\mathbf{(x_1 - x_2)^2 + (y_1 - y_2)^2 = d^2}$

Areas

Trapezoid:
Area = ½ ($base_1 + base_2$) x height

Law of Sines

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

$$a = b \times \frac{\text{Sin A}}{\text{Sin B}} \qquad a = c \times \frac{\text{Sin A}}{\text{Sin C}}$$

$$b = a \times \frac{\text{Sin B}}{\text{Sin A}} \qquad b = c \times \frac{\text{Sin B}}{\text{Sin C}}$$

$$c = a \times \frac{\text{Sin C}}{\text{Sin A}} \qquad c = b \times \frac{\text{Sin C}}{\text{Sin B}}$$

$$\text{Sin A} = \frac{a \times \text{Sin B}}{b} \qquad \text{Sin A} = \frac{a \times \text{Sin C}}{c}$$

$$\text{Sin B} = \frac{b \times \text{Sin A}}{a} \qquad \text{Sin B} = \frac{b \times \text{Sin C}}{c}$$

$$\text{Sin C} = \frac{c \times \text{Sin A}}{a} \qquad \text{Sin C} = \frac{c \times \text{Sin B}}{b}$$

Law of Cosines

$\mathbf{c^2 = a^2 + b^2 - (2\,ab\,\text{Cos C})}$

$\mathbf{b^2 = a^2 + c^2 - (2\,ac\,\text{Cos B})}$

$\mathbf{a^2 = b^2 + c^2 - (2\,bc\,\text{Cos A})}$

$$\text{Cos A} = \frac{(b^2 + c^2) - a^2}{2\,bc}$$

$$\text{Cos B} = \frac{(a^2 + c^2) - b^2}{2\,ac}$$

Triangle:
Area = ½ (b x h)

Curves

Radius

$$R = \frac{\text{T(angent length)}}{\tan(\tfrac{1}{2}\,\text{Delta}\Delta)} = \frac{180 \times \text{ArcL}}{\pi \times \text{Delta}\Delta}$$

Degree of Curve

$$D_c = \frac{5729.58}{R}\ \text{(Hwy)} \qquad D_c = \frac{5729.65}{R}\ \text{(RR)}$$

Arc

$$\text{Arc} = \frac{100 \times \text{Delta}\Delta}{D_c}$$

Arc = 100 x (DeltaΔ ÷ Dc)

Chord

chord = 2R x (sin ½ DeltaΔ)

Delta or Central Angle Δ

$$\text{delta}\Delta = \frac{\text{ArcL}}{R} \times 5729.58$$

Tangent Length

tangent = R x (tan ½ Δ)

Deflection Angle

Deflection angle = ½ DeltaΔ

Sector Area

Sector area = (πR^2) * (Δ ÷ 360)

Segment Area

Segment area = sector area - $\frac{R^2}{2}$ * sin Δ

Fillet

Fillet = R x T – (πR^2) * (Δ ÷ 360)
Fillet = R x T – sector

PC Station =
PI station – tangent length

PT Station =
PC station + arc length

$$\text{Cos } C = \frac{(a^2 + b^2) - c^2}{2ab}$$